1. Construct a deterministic finite automaton which is equivalent to the following non-deterministic finite automaton using the algorithm we have learned in class. \( \Sigma = \{a, b\} \)
2. (a) Define the star operation on languages. (If \( A \) is a language then what is the definition of the language \( A^* \)?)

(b) Prove that if \( A \) is a regular language then \( A^* \) is also regular. (We have proved this in class.)
3. Let $\Sigma = \{a, b\}$ and let the language $L$ contain all words over $\Sigma$ for which at least one of the following conditions are satisfied:

- the number of $a$ characters and the number of $b$ characters in the word are both odd
- the first and last character of the word is the same
- the word does not contain the subword $bbbb$

Prove that $L$ is a regular language.
4. Prove that the language $L = \{(ab)^n c^{5n} \mid n \geq 1\}$ is not regular. Use the pumping lemma.