Combinatorial Optimization:
Retake of the first midterm exam 2017.04.20.

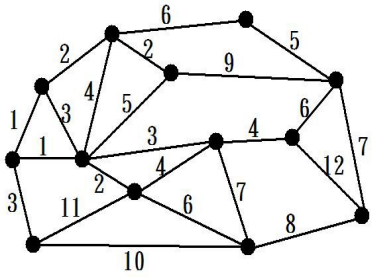
Please indicate your name and Neptun code on top of your test! Every problem is worth 10 points, the minimum number of points required for passing is 24. You can work for 90 minutes. You are not allowed to use written or printed materials, a calculator or any other aid when writing the test. Please make sure to justify your answers - unjustified results are worth no points. I wish you all the success!

1. Prove that the Hamiltonian cycle problem is in NP. (Hamiltonian cycle problem: Input: G graph. Output: True if and only if there exists a cycle in G which goes through each vertices exactly once.)

2. Give a polynomial reduction from the 4-coloring problem to the 5-coloring problem. (k-coloring problem: Input: G graph. Output: True if and only if there exists a coloring of the nodes of G with k different colors, such that no two vertices sharing the same edge have the same color.)

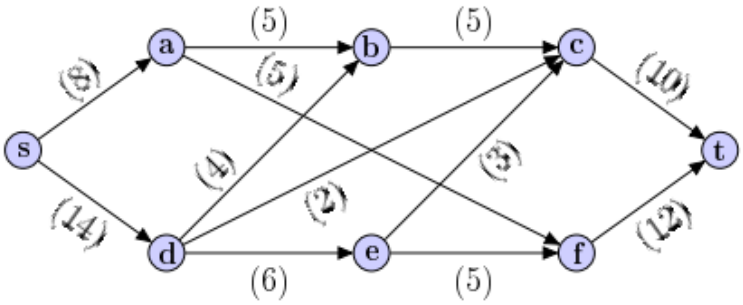
3. Find a maximum matching in the graph represented by this matrix: This graph has 16 vertices, a_1, a_2,...a_8,b_1, b_2,...b_8, and there is an edge between a_i and b_j if and only if the j-th element of the i-th row is 1. (Use the alternating path method)

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$



4. Run the Kruskal method to find a minimum spanning tree in the graph on the left. (Indicate the order of the decisions)

5. Find a maximum flow and a minimum cut with the Ford-Fulkerson method in the graph on the right. (It starts from s and the target is t)



6. Define NP-completeness.