

## Decidable and recognizable languages

1. Prove that the language of all prime numbers (in binary representation) is recursive (Turing-decidable). (You can use the fact, that arithmetic operations can be performed by Turing machines.)
2. Prove that
  - (a)  $\overline{L_d} \notin \text{R}$
  - (b)  $\overline{L_d} \in \text{RE}$
3. Let  $L = \{w\#s : w \in L_d, \text{ and } s \notin L(M_w)\}$ . Prove that  $L$  is undecidable.
4. Let  $L$  be the language of descriptions  $w$  of Turing machines where  $M_w$  accepts the prefix of  $w$  of length 5 (i.e. the string formed by the first 5 characters of  $w$ ). Show that this language is recognizable (it is in RE).
5. Let  $L = \{w : w \text{ is a description of a TM and } L(M_w) = \emptyset\}$ .
  - (a) Prove that  $L$  is undecidable.
  - (b) Is it true that  $L \in \text{RE}$ ?
  - (c) Is it true that  $L \in \text{coRE}$ ?
6. The language  $L$  consists of Turing machine descriptions  $w$ , such that  $M_w$  accepts only strings of odd length (but not necessarily accepts all of these strings). Prove that
  - (a)  $L$  is not decidable.
  - (b)  $\overline{L}$  is recognizable.
7. A Turing machine without left move is like an ordinary TM except, that it can never move its head to the left (but it is allowed to move its head to the right or do not move it). Show that this variant is not equivalent to the original model. What class of languages do these machines recognize?