

Turing machines

1. Give the state diagram of a Turing machine which recognizes $L = \{w\#w \mid w \in \{0,1\}^*\}$.
2. Give the state diagram of a Turing machine which recognizes $L = \{w\#w^R \mid w \in \{0,1\}^*\}$, where w^R is the reverse of w .
3. Let M be the following 2-tape Turing machine: $M = (Q, \Sigma, \Gamma, \delta, *, q_0, F)$, where $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$, $\Sigma = \{0,1\}$, $\Gamma = \{X, 0, 1, *\}$, $F = \{q_5\}$,

$\delta :$

	reading		writing				
state	1st tape	2nd tape	1st tape		2nd tape		new state
q_0	0	*	0	S	X	R	q_1
	1	*	1	S	X	R	q_1
	*	*	*	S	*	S	q_5
q_1	0	*	0	R	0	R	q_1
	1	*	1	R	1	R	q_1
	*	*	*	S	*	L	q_2
q_2	*	0	*	S	0	L	q_2
	*	1	*	S	1	L	q_2
	*	X	*	L	X	R	q_3
q_3	0	0	0	S	0	R	q_4
	1	1	1	S	1	R	q_4
q_4	0	0	0	L	0	S	q_3
	0	1	0	L	1	S	q_3
	1	0	1	L	0	S	q_3
	1	1	1	L	1	S	q_3
	0	*	0	S	*	S	q_5
	1	*	1	S	*	S	q_5

Which 0–1 sequences are accepted by M ? (By our definition if δ is not defined and the current state is not in F , then M rejects.)

4. Let the language $L \subset \{a, b\}^*$ contain the strings s that start with a , and in s the i th block of letters is at least i . (For example $abbbbbaaa \in L$, because the first block has length 1, the second has length $4 \geq 2$, the third has length 3.) Design a Turing machine M that accepts L . You can use more than one tape if it is convenient. (Describe in English the idea how your machine works, then specify the TM according to the definition or by a diagram.)
5. Let L_1, L_2 be two languages. Prove that $L_1, L_2 \in R$ implies that $L_1 \cup L_2 \in R$.