Gallai colorings and domination in multipartite digraphs

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8th French Combinatorial Conference, Paris, 2010.

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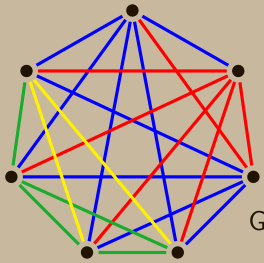
Gallai coloring of a graph

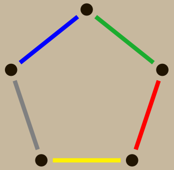
An edge-coloring of a graph is called a **Gallai coloring** if there is no completely multicolored triangle.

A Gallai colored complete graph has a color class which spans a connected subgraph on the entire vertex set.



Gallai colored K_7

Gallai coloring of a graph



Gallai colored C_5

Theorem (Gyárfás, Sárközy [GyS[4]])

In a Gallai coloring of a graph G there is a monochromatic component with at least $\frac{|V(G)|}{\alpha^2(G)+\alpha(G)-1}$ vertices, where $\alpha(G)$ is the independence number of G.

[4] [GyS] A. Gyárfás, G. N. Sárközy, Gallai colorings of non-complete graphs, Discrete Mathematics

Gallai coloring of a graph

Suppose that the edges of a graph G are colored so that no triangle is colored with three distinct colors. Is it true that the vertices of G can be covered by the vertices of at most k monochromatic components where k depends only on the independence number of G?

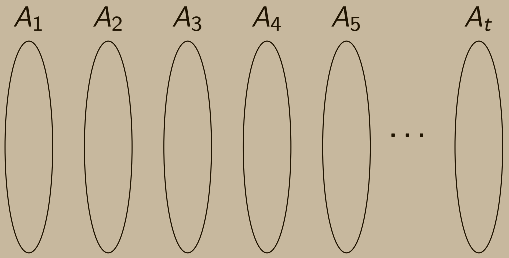
Theorem 1 (Gyárfás, Simonyi, Tóth)

Given a Gallai coloring of a graph G the vertices of G can be covered by the vertices of at most $g(\alpha(G))$ monochromatic components.

Special case: In case $\alpha(G) = 2$ at most 5 components are enough.

Domination of multipartite digraphs

Let D be a **multipartite digraph** (i.e., its vertices are partitioned into classes $A_1, \ldots, A_t$ of independent vertices) **without cyclic triangles**.



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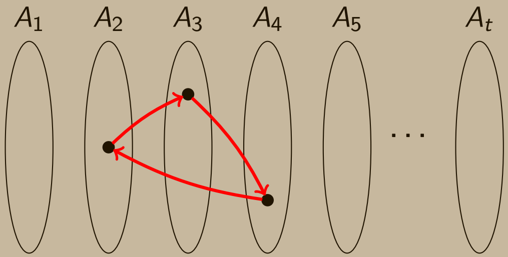

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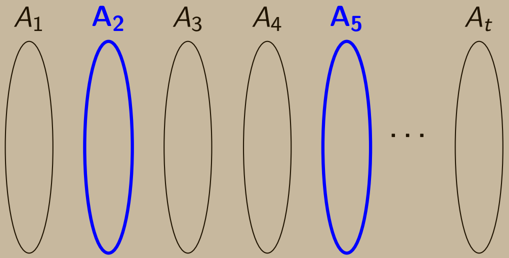

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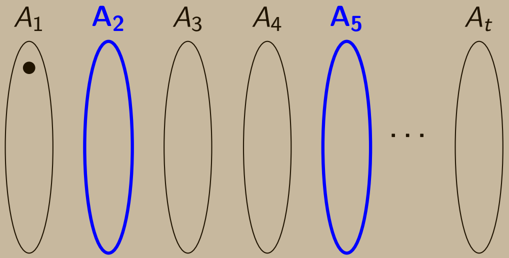

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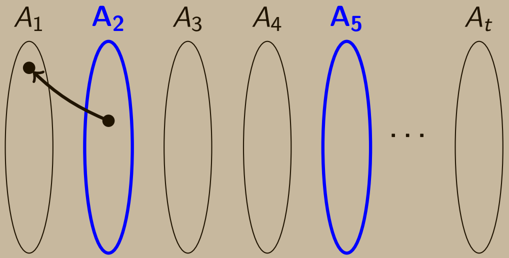

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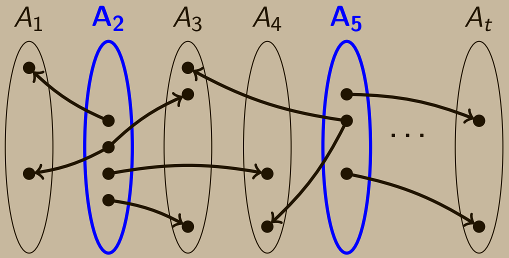

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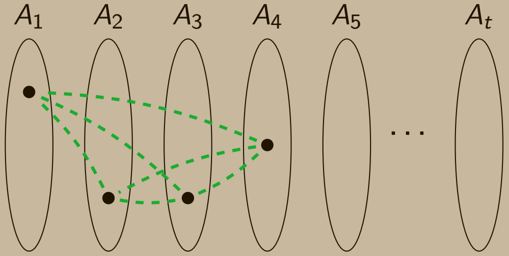

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Theorem 2 (Gyárfás, Simonyi, Tóth)

There exists a $h = h(\beta(D))$ such that D has a dominating set of size at most h.

Special case: In case $\beta(D) = 1$ there is a dominating set of size 1. In case $\beta(D) = 2$ there is a dominating set of size at most 4.

Proof of Theorem 1 from Theorem 2

Theorem 1. G Gallai colored graph, $\alpha(G) = 2 \Rightarrow$ the vertices of G can be covered by the vertices of at most 5 monochromatic components.

Theorem 2. D multipartite digraph, no cyclic triangle, $\beta(D) = 2 \Rightarrow$ there is a dominating set of size at most 4.

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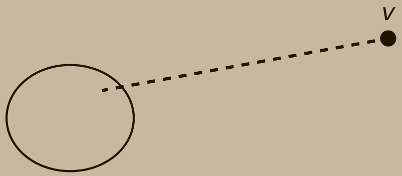

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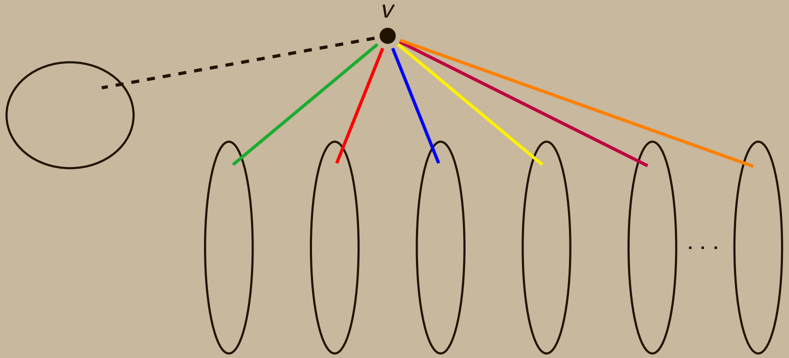

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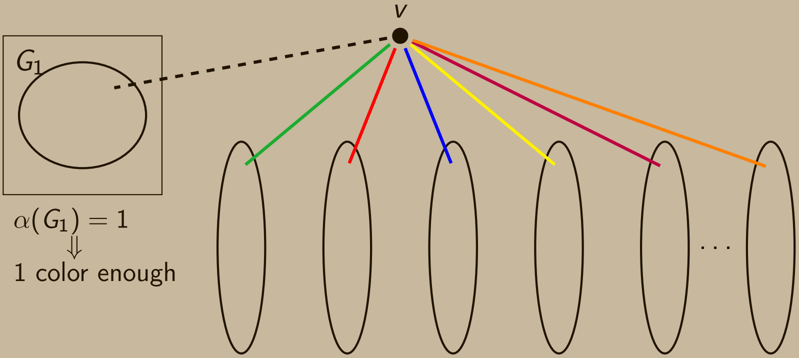

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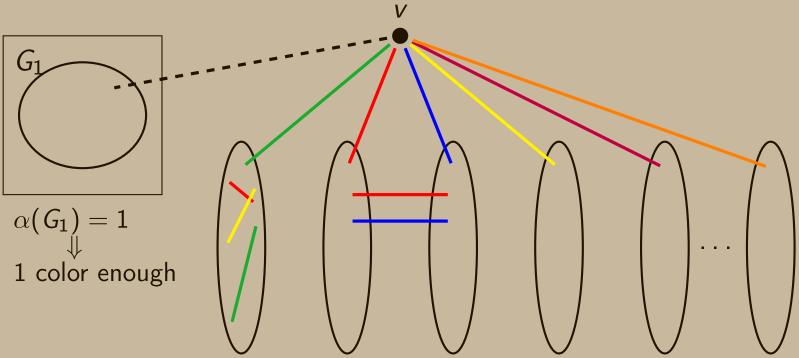

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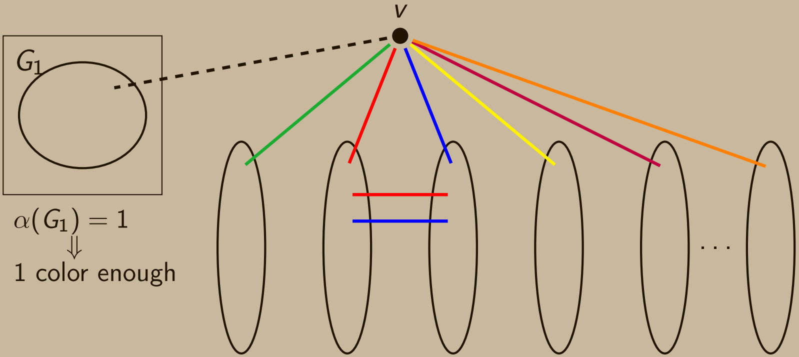

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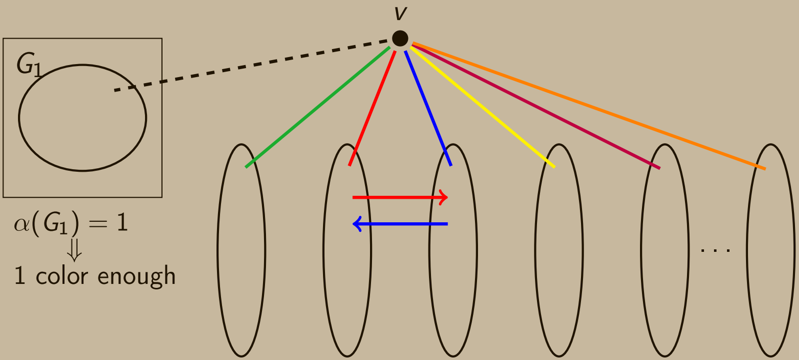

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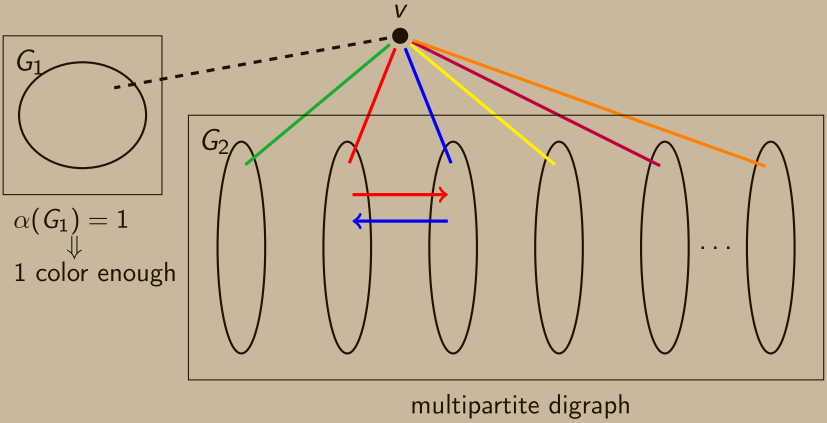

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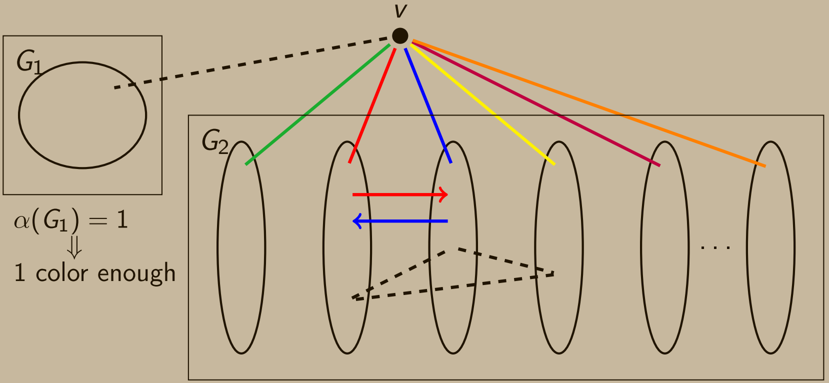

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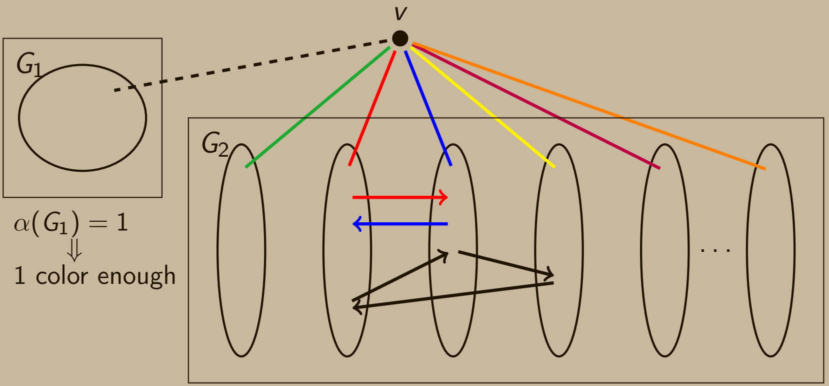

$\alpha(G_1) = 1$
$\Downarrow$
1 color enough

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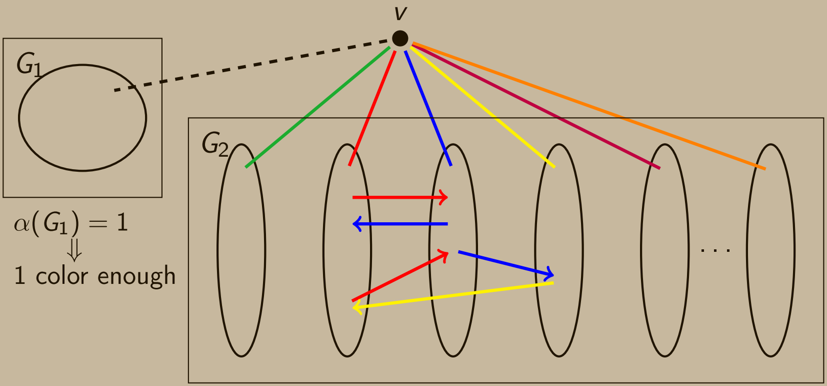

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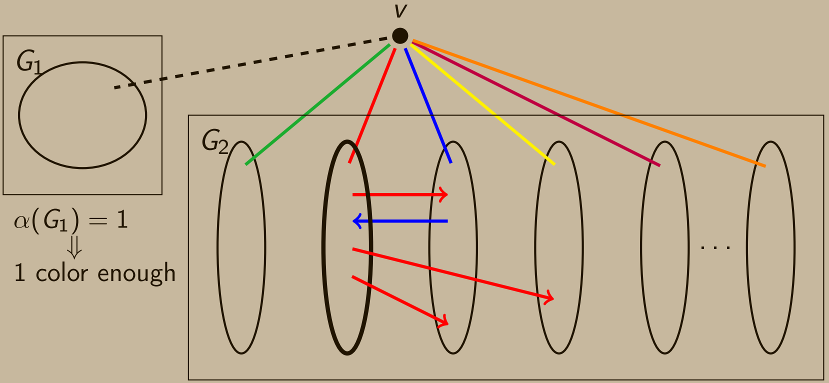

4 color enough $\Leftarrow$ multipartite digraph
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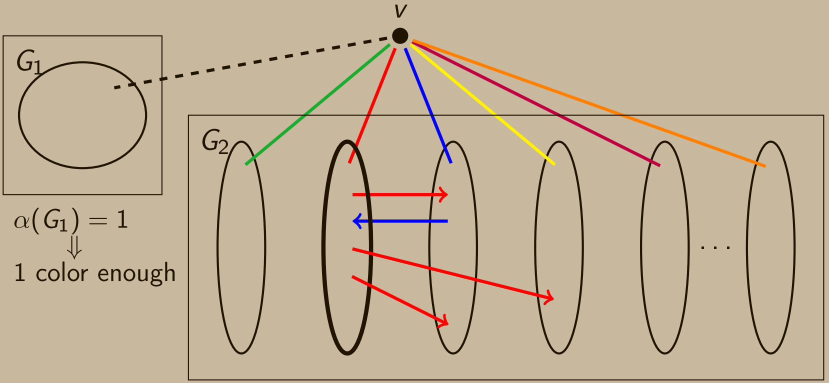


q.e.d. 4 color enough $\Leftarrow$ multipartite digraph
no 3-sized transversal independent set
no cyclic triangle

Proof of Theorem 2 (special case)

Theorem 2 (in the case $\beta(D) = 2$)
Let D be a multipartite digraph with no cyclic triangle and $\beta(D) = 2$
$\Rightarrow$ there is a dominating set of size at most 4.

Lemma 3 (Gyárfás, Simonyi, Tóth)

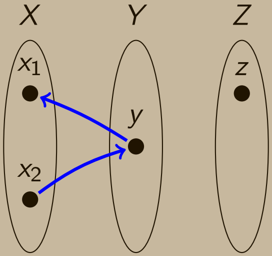
*Let D be a multipartite digraph with no cyclic triangle and $\beta(D) = 1$.
$\Rightarrow$ There is a partite class K which is a dominating set, and there is a vertex $k \in K$ such that k dominates all the vertices of $V(D) \setminus (K \cup L)$ for some partite class $L \neq K$.*

Proof of Lemma 3.

D is a multipartite digraph with no cyclic triangle and $\beta(D) = 1$.

Observation 4

Suppose that for vertices $x_1, x_2 \in X$ and $y \in Y$ the edges (x_2, y) and (y, x_1) are present in D. Then for every $z \in Z \neq X, Y$ with $(x_1, z) \in E(D)$ we also have $(x_2, z) \in E(D)$.



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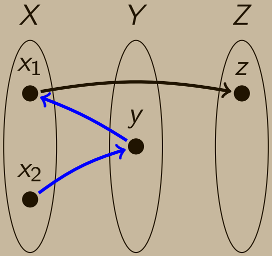

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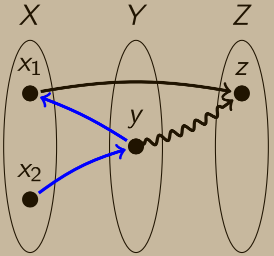

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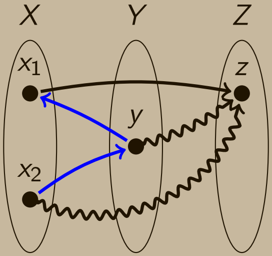

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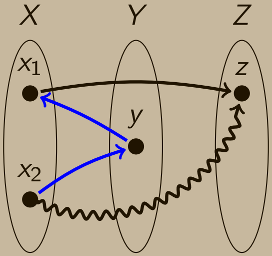

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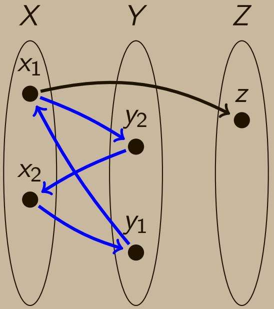
Suppose that for vertices $x_1, x_2 \in X$ and $y_1, y_2 \in Y$ the edges (x_1, y_2), (y_2, x_2), $(x_2, y_1), (y_1, x_1)$ are present in D forming a cyclic quadrangle. Then these four vertices split every partite class $Z \neq X, Y$ in the same way (they have the same out- and inneighbourset).



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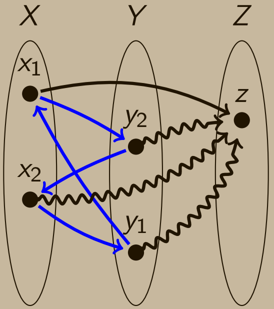

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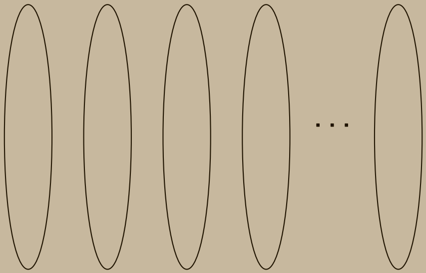

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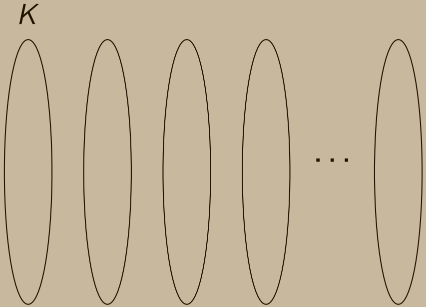

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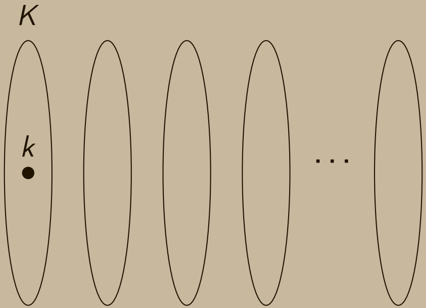

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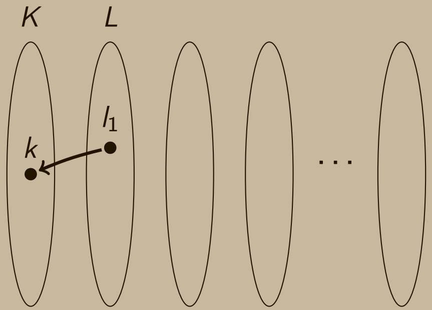

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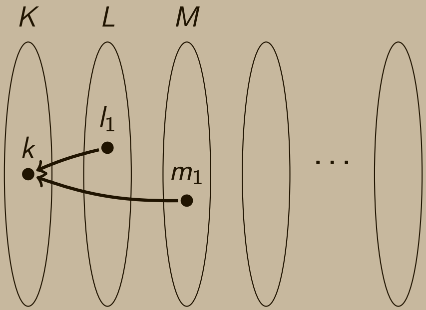

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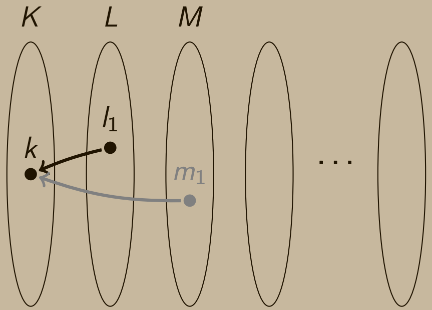

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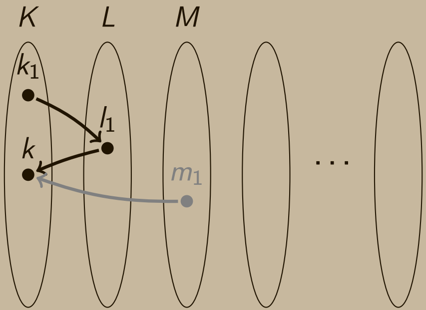

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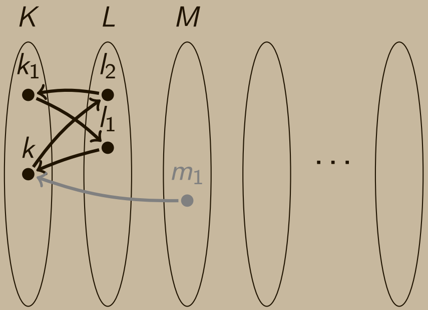

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Let k be an element of K which has the most outneighbours.

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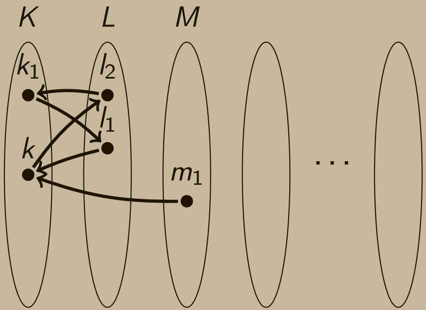

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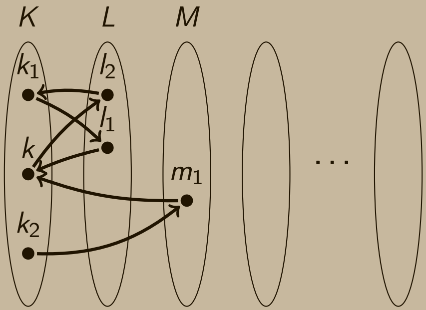

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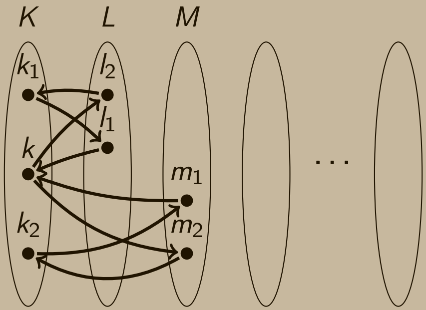

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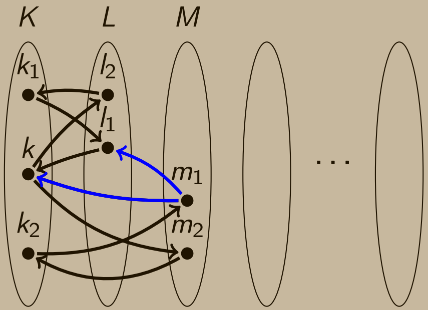

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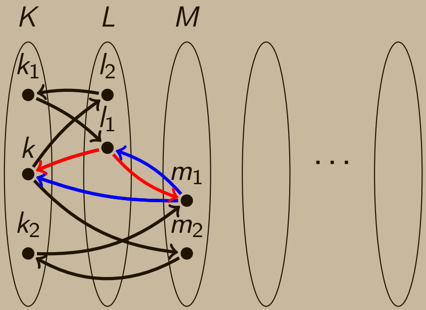

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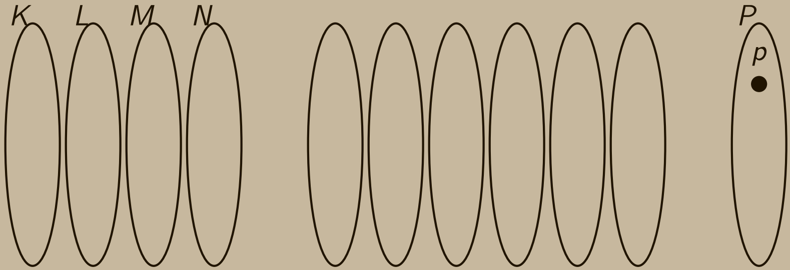
Proof of Theorem 2.

Theorem 2 (in the case $\beta(D) = 2$)
Let D be a multipartite digraph with no cyclic triangle and $\beta(D) = 2 \Rightarrow$ there is a dominating set of size at most 4.

K, L, M and N form a dominating set of $D \setminus p$.



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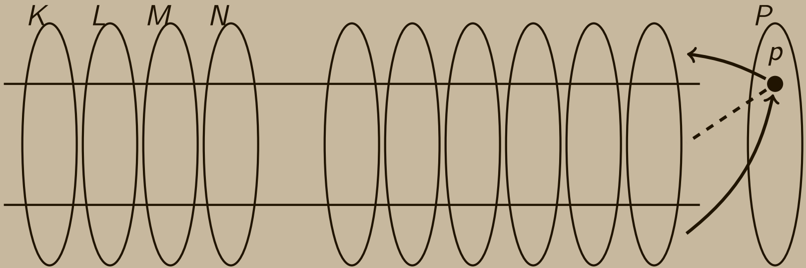

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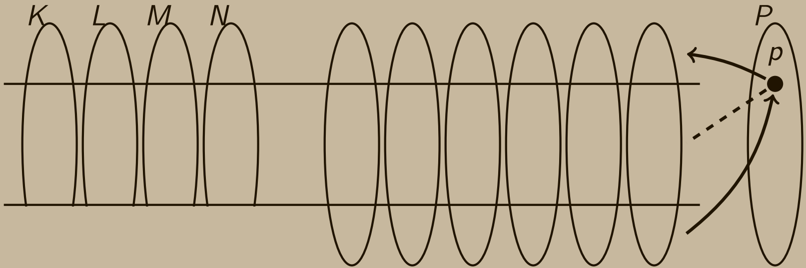

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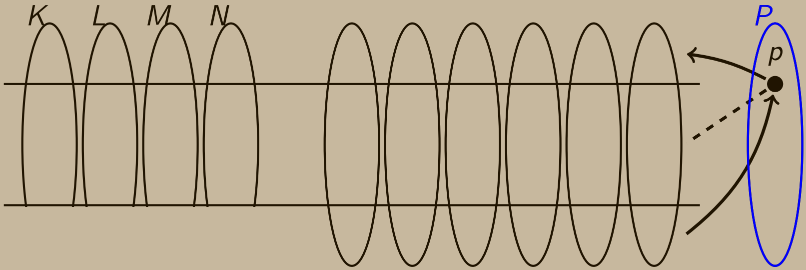

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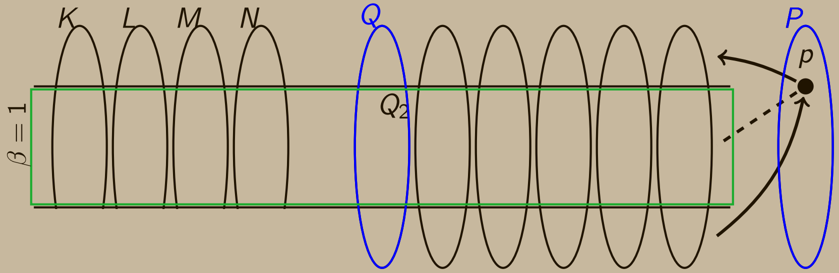

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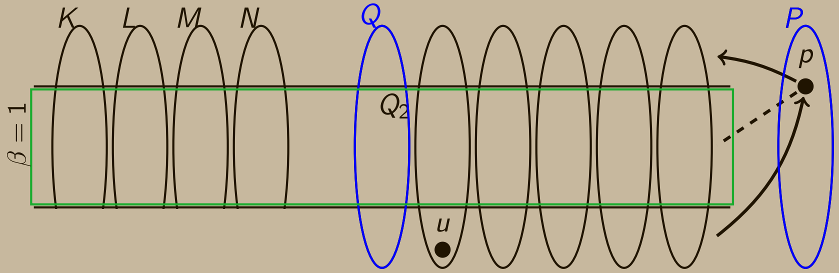

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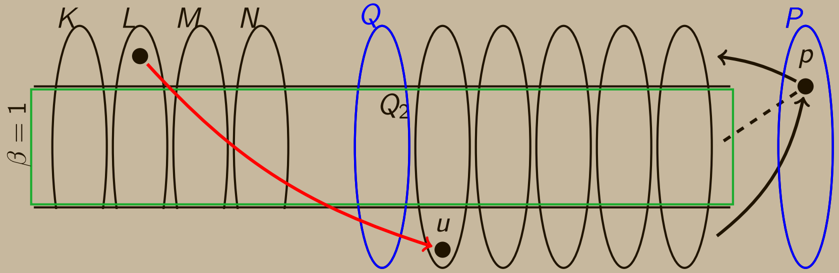

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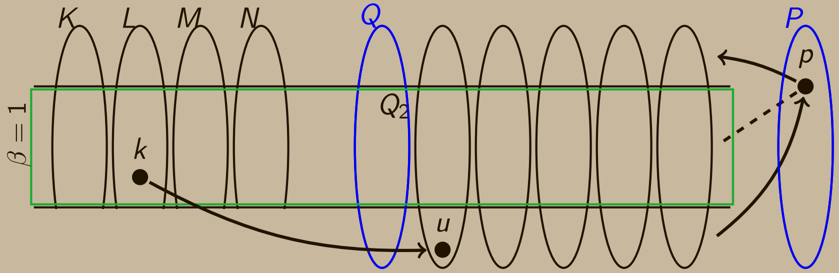

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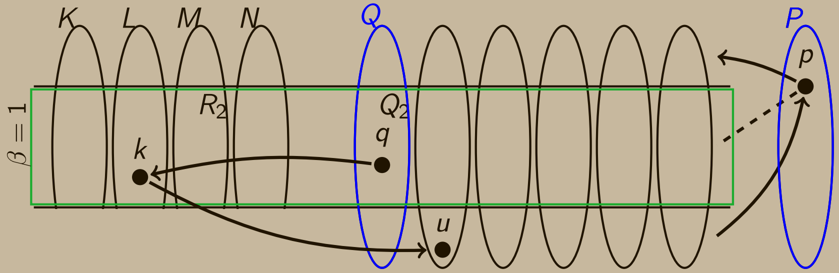

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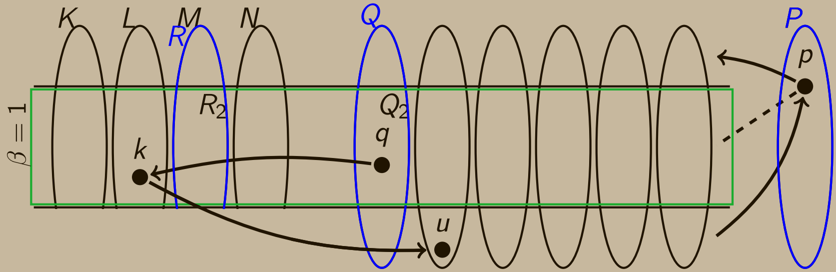

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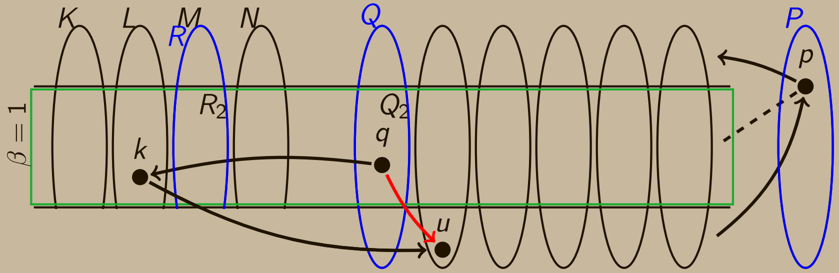

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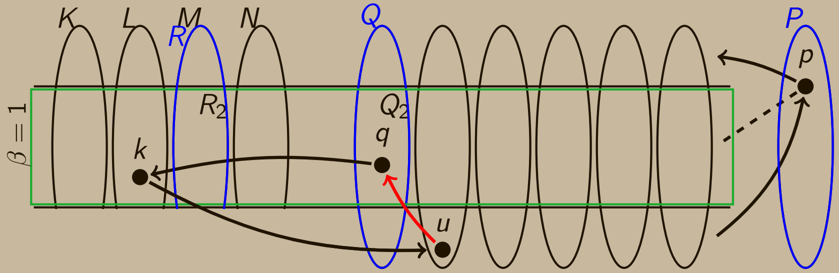

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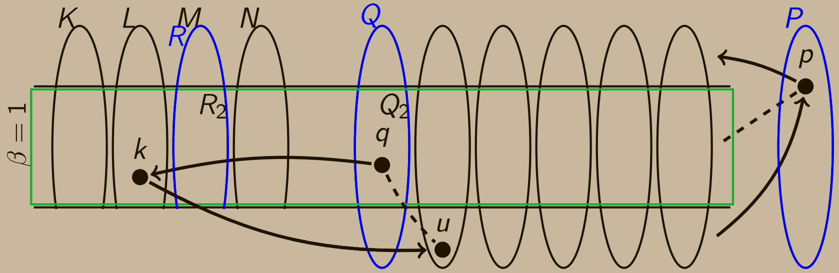

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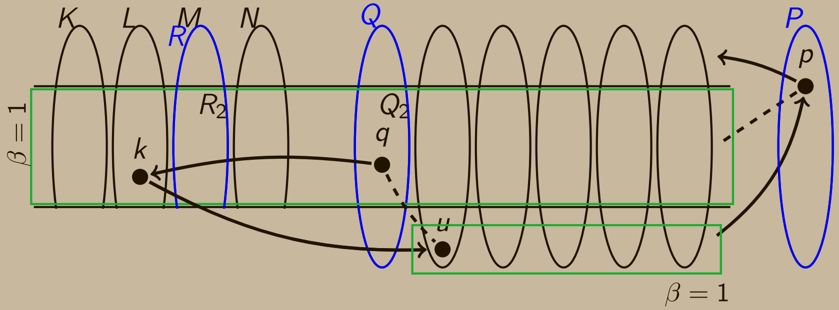


We create a dominating set of D: P, Q, R, S.

Remarks

Theorem 2. For every integer β there exists an integer $h = h(\beta)$ such that the following holds. If D is a multipartite digraph such that D contains no cyclic triangle and $\beta(D) = \beta$, then D has a dominating set of size h.

We can state a little bit more: There is a set of at most $h_1(\beta)$ vertices of D which dominates the whole graph except perhaps their own partite classes and at most $h_2(\beta)$ other exceptional classes. From the proof we obtain the recursion formula

$$h_1(\beta) \leq 2\beta + (2\beta + 1)h_1(\beta - 1)$$

and

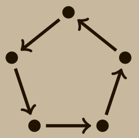
$$h_2(\beta) \leq \beta + (2\beta + 1)h_2(\beta - 1).$$

Remarks

An important special case is when $|A_i| = 1$ *for each* $i \in [t]$. In this case $\beta(D) = \alpha(D)$ and we want to estimate $\gamma(D)$, the usual domination number of D, the smallest number of vertices in D whose closed outneighborhoods cover $V(D)$.
The class of digraphs with no directed triangles, is studied already and called the class of **clique-acyclic digraphs**.

Theorem 6

Let $f(1) = 1$ *and for* $\alpha \geq 2$, $f(\alpha) = \alpha + \alpha f(\alpha - 1)$. *If* D *is a clique-acyclic digraph then* $\gamma(D) \leq f(\alpha(D))$.
Special case: If $\alpha(D) = 2$, *then* $\gamma(D) \leq 3$.



Thank you for your attention!