IMPROVING SIZE-BOUNDS FOR SUBCASES OF SQUARE-SHAPED SWITCHBOX ROUTING*

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Abstract

Various problems are studied in the field of detailed VLSI routing. According to the position of the terminals on the outer boundaries of a rectangular grid, one can distinguish between different subcases of Switchbox Routing: C-shaped, Gamma, Channel and Single Row Routing.

This paper focuses on the minimum number of layers needed for solving these problems either in the Manhattan, or in the unconstrained wiring model. Besides summarizing earlier known bounds, our new results show examples for C-shaped routing problem instances of arbitrary size that cannot be solved using 3 layers in the Manhattan, or 2 layers in the unconstrained model, respectively.

keywords: VLSI-design, Switchbox routing, layers, rectangular grid

I. Introduction

Several different models are known for the detailed routing phase of VLSI-design [5], namely, for the case when the position of pins to be interconnected is given already, and the connections must be established using a minimal size cubic grid.

Regarding either the position of the pins, or the routing model of the layers, we can distinguish between different problems and models. Although these problems are under intensive research, the exact number of layers needed is still not known for most cases.

In this paper we focus on routing problems having square-shaped layers. We summarize the earlier results, upper and lower bounds given for the different subcases of the Switchbox Routing problem. Some results are improved to decrease the gap between the bounds.

II. Basic definitions

A *switchbox* is a rectangular grid G consisting of horizontal *tracks* (numbered from 0 to $w+1$) and vertical *columns* (numbered from 0 to $h+1$), where w is the *width* and h is the *height* of the switchbox. In this paper, we assume *square-shaped* layers, namely $w = h$. The boundary points of G are called *terminals*. Depending on which boundary of the switchbox they are situated on, the terminals are called *Northern*, *Southern*, *Eastern* or *Western*. The "corners" of the switchbox are not regarded as terminals, routings must not use them either.

A *net* is a collection of terminals. A *switchbox routing problem* is a set $\mathcal{N} = \{N_1, \ldots, N_n\}$ of pairwise disjoint nets.

The *solution* of a routing problem is a set $\mathcal{H} = \{H_1, \ldots, H_n\}$ of pairwise vertex disjoint subgraphs (also called *wires*, these are usually Steiner trees) of the k–layer rectangular grid such that H_i connects the terminals of N_i for $i = 1, \ldots, n$. The wires can access the terminals on any layer. Edges of the wires that join adjacent vertices of two consecutive layers are called *vias*.

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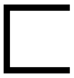
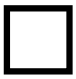
	Manhattan model	Unconstrained model
	$= 2$ [2]	$= 2$ [2, 7, 12]
	$= 3$ [2, 4]	≥ 2 ≤ 3 [6, 7, 10]
	≥ 3 [1, 12] ≤ 4	≥ 2 ≤ 3 [12]
	$\geq \mathbf{4}$ ≤ 5 [11]	$\geq \mathbf{3}$ ≤ 5 [11]
	$= 6$ [3, 11]	≥ 4 [1] ≤ 6[11]

Table 1: Known bounds for the minimum number of layers in different square-shaped routing problems

A specific routing problem can be solved either in the *unconstrained k–layer model*, or in the *Manhattan model*.

In the *unconstrained k–layer model* no further restrictions are applied to layers or to subgraphs H_i. Contrarily, in the multilayer *Manhattan model* consecutive layers must contain wire segments of different directions. Thus, in this model, layers with horizontal (East–West) and with vertical (North–South) wire segments alternate.

According to the position of terminals, several subcases of the switchbox routing problem are considered, as follows:

- Single Row Routing problem, if all terminals are Northern;
- Channel Routing problem, if all terminals are either Southern or Northern;
- Γ–Routing problem, if all terminals are either Northern or Western;
- C–Routing problem, if all terminals are either Southern or Western or Northern;
- General Switchbox problem, if terminals are situated on all four boundaries of the grid.

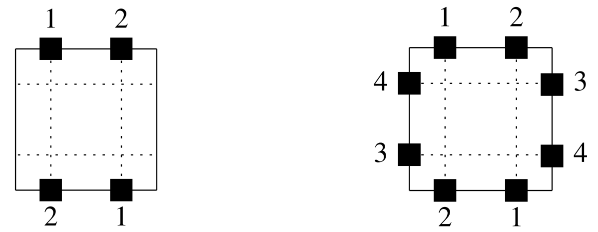
III. Square-shaped switchbox routing

In this paper, a survey is given on the results concerning the number of layers needed in both models. Table 1 shows the different routing problems and gives the most important references. Our new results on C-shape routing are printed in bold face letters.

Note that the figures in the table show the minimum number of layers that enables to solve *any* routing problem. Hence, some specific problems can be solved using fewer layers than the corresponding universal bound. On the other hand, obviously, any upper bound in the Manhattan model is an upper bound in the Unconstrained model, and any lower bound in the Unconstrained model is a lower bound in Manhattan model.

A simple algorithm for Single Row Routing in the 2–layer Manhattan model is based on the fundamental studies of Gallai [2]. Further results on the effect of using more layers are given in [8], while finding the minimum wire-length solution turned out to be NP-hard [12].

Channel Routing problems cannot always be wired on two layers in the Manhattan model, as Fig. 1(a) shows an example. Note that this example can be extended to an arbitrary sized one. However, three layers are always enough in the Manhattan model [2, 4], and even two layers can be enough in the Unconstrained model [6, 7, 10].



(a) $\geq$ 3 layers are needed for Channel Routing

(b) $\geq$ 6 layers are needed for Switchbox Routing

Figure 1: Examples where lower bounds are sharp in the Manhattan model

Wiring a Gamma Routing problem in the Manhattan model can require at least three layers [12], and can be trivially solved on four layers in the Manhattan model.

Gamma Routing problems are always solvable on three layers in the Unconstrained model. Furthermore, the following theorem can be stated as a straightforward consequence of the method in [12].

Theorem 1 *Every Gamma Routing problem can be solved in the Unconstrained model using three layers such that one of the layers is used by a single net only.*

Szeszlér [11] gave an efficient linear time algorithm to solve the general Switchbox problem in the Manhattan model using at most $2\lceil m \rceil + 4$ layers.

This yields a 6-layer solution for every square-shaped switchbox routing problem. Moreover, the construction includes a 5-layer solution for all C-shaped routing problems where $w = h$.

As the problem of Fig. 1(b) cannot be solved on five layers [3], this bound is sharp. Note that for this case, only a small size example is known, unlike in the case of Gamma Routing, where we have examples of arbitrary size. In the Unconstrained model, no such example is known, but a lower bound of four layers is already proved.

As mentioned above, Szeszlér's algorithm gives a trivial way to solve C-shaped Routing problems on five layers. Our new results give a lower bound of four layers in the Manhattan model and a lower bound of three layers in the Unconstrained model as stated in the following theorems:

Theorem 2 *For all $w \geq 3$ there exists a C-shaped Routing problem on the $w \times w$ grid which cannot be solved on three layers in the Manhattan model.*

Proof is based on the routing problem in Fig. 2(a), and given in details in [9].

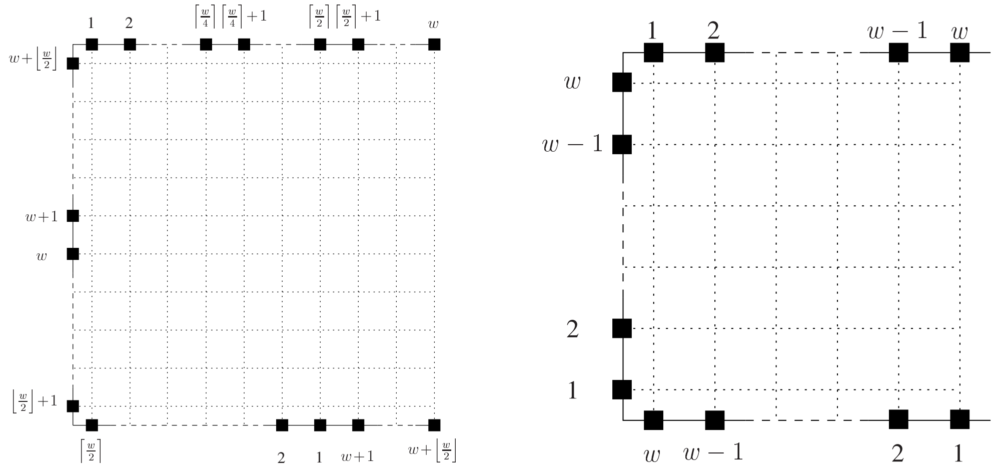
Theorem 3 *For all $w \geq 2$ there exists a C-shaped Routing problem on the $w \times w$ grid which cannot be solved on two layers in the Unconstrained model.*

Proof is based on the routing problem in Fig. 2(b), and given in details in [9].

IV. Summary

Our new results improve lower bounds for the number of layers needed to solve any C-shaped Routing problem. Namely, we gave two problem instances that cannot be solved using less than 4 or 3 layers in the Manhattan or in the Unconstrained model, respectively. Both can be extended to an arbitrarily large size.

However, for 6 problems out of 10 mentioned here (see Table 1) the exact number of layers needed is still not known; this can be a subject of further research.



(a) $\geq$ 4 layers are needed in Manhattan model (b) $\geq$ 3 layers are needed in Unconst. model

Figure 2: Examples where lower bounds are sharp in the Manhattan model

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