Some polynomially solvable subcases of the detailed routing problem in VLSI design

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Abstract: There are plenty of **NP**-complete problems in VLSI design, like channel routing or switchbox routing in the 2-layer Manhattan model (2Mm, for short). However, there are quite a few polynomially solvable problems as well. Some of them (like the single row routing in 2Mm) are "classical" results; in a past survey [36] we presented some more recent ones, including

1. a linear time channel routing algorithm in the unconstrained 2-layer model;

2. a linear time switchbox routing algorithm in the unconstrained multilayer model; and

3. a linear time solution of the so called *gamma routing problem* in 2Mm.

(This latter means that all the terminals to be interconnected are situated at two *adjacent* sides of a rectangular routing area, thus forming a Γ shape. Just like channel routing, it is a special case of switchbox routing, and contains single row routing as a special case.)

In the present survey paper we also mention some results from the last three years, including

4. some negative results (**NP**-completeness) in the multilayer Manhattan model and a channel routing algorithm if the number of layers is even;

5. an interesting relation between channel routing and multiprocessor scheduling; and

6. some improvements of **1.-2.-3.** above.

We present the (positive and negative) results in a systematic way, taking into account two hierarchies, namely that of geometry (what to route) and technology (how to route) at the same time.

Keywords: VLSI design, detailed routing, channel routing

1 A hierarchy of geometric patterns

Within the complex problem of designing VLSI (Very Large Scale Integrated) circuits we consider the phase of *detailed routing* only, that is, the position of the devices to be interconnected is given already and certain pins of these devices must be interconnected by wires. We consider a rectangular portion of a square grid (in case of planar routing) or a 3-dimensional cubic grid consisting of k parallel levels (in case of k-layer routing). This 2- or 3-dimensional grid-portion is considered as a graph G. The four sides of the rectangle in case of $k = 1$ or the k copies of them in case of $k \geq 2$ will be called the *boundary* of G. If $k \geq 2$ then we may leave a grid point of a certain level for another grid point of the adjacent level along a grid edge; such a step is called the application of a *via*.

The points of the graph corresponding to the pins of the devices are called *terminals*. If a set of terminals have to be interconnected, this set is called a *net*. A *routing problem* is a collection of nets. If each net contains at most p terminals, we speak about a *p-terminal routing problem* (see

Remark 1 as well.) A *solution* of a routing problem is a collection of vertex-disjoint (see Remark 2 as well) connected subgraphs (usually Steiner trees) of G so that two terminals are in the same subgraph if and only if they belong to the same net.

We always suppose, moreover, that the terminals are at the boundary (excluding the corners) of G and that the subgraphs do not contain edges of the boundary. If $k \geq 2$ then the subgraphs may leave the terminal at any layer; one can imagine via holes at the boundary of G.

If the terminals may appear at all the four sides of the boundary then the routing problem is called *switchbox routing.* If they arise on two parallel sides only then we speak about *channel routing.* If a single side is used only, this is the *single row routing problem.*

Beside these classical concepts, there are two possibilities: The *gamma routing problem* means that all the terminals are situated at two adjacent sides, thus forming a Γ shape. Finally, if three sides are used then we speak about an *open switchbox.*

Clearly, both channel routing and gamma routing are special cases of (usual and open) switchbox routings, and both contain single row routing as a special case.

In case of single row routing the single side containing the terminals will be called the Northern side. In case of channel and gamma routings the corresponding two sides will be called Northern and Southern, and Northern and Western, respectively. The horizontal grid lines are usually called *tracks,* the vertical ones are called *columns.* The vertical dimension (distance of the Northern and Southern sides measured by the number of the tracks) is called the *width* of the routing problem and will be denoted by w. The horizontal dimension (distance of the Western and Eastern sides measured by the number of the columns) is called the *length* of the routing problem and will be denoted by l.

Remark 1. A frequently studied special case is the *bipartite channel routing* when each net consists of two terminals – one in the Northern and one in the Southern boundary. Several authors call this case the 2-terminal channel routing problem. By our definition this latter allows North-only or South-only nets as well.

In case of single row routing and channel routing problems the routing area may or may not be prescibed. In the latter case we have a minimization problem: Only the length is given and find a solution with minimum width. In the former case and in case of gamma and (usual and open) switchbox routings both the length and the width are given and we have a decision problem: Decide whether a solution exists. However, see Remark 4 below as well.

2 A hierarchy of technologies

If all the interconnections have to be realized in a single layer ($k = 1$, see above) then we speak about *planar routing,* otherwise about *multilayer (or k-layer) routing.*

Within 2-layer routing *Manhattan routing* has been studied the most widely: this means that one of the layers is reserved for horizontal wire segments and the other for vertical ones, hence each 90 degree turn requires a via. This concept can easily be extended for $k > 2$ layers as well: in this case layers containing horizontal wire segments only and layers, containing vertical wire segments only, are alternating.

Another, weaker restriction within 2-layer routing is the *knock knee routing* where a 90 degree turn within the same layer is possible. Wires in different layers can, therefore, cross or share corners (i.e. knock knees) but are not allowed to overlap for any distance.

If no restriction is given, we shall frequently call the k-layer routing (for $k \geq 2$) *unconstrained* as well.

Remark 2. We defined the solution of the routing problem as a collection of vertex-disjoint subgraphs. Several authors have studied those "solutions" of various routing problems where a single layer is given but the subgraphs need to be edge-disjoint only. This will be called *single layer (or planar) edge-disjoint routing.* This research started with the pioneering paper of Frank [15], see also [30] and the surveys [32], [49].

Observe that if a routing problem can be solved in two layers using either the Manhattan or the knock knee model then the projection of the two layers automatically gives a single layer edge-disjoint solution. Hence single layer edge-disjoint routing (just like 2-layer unconstrained routing) contains Manhattan and knock knee routings as special cases. On the other hand, single layer edge-disjoint routing (just like 2-layer unconstrained routing) is a special case of multilayer routing since any edge-disjoint solution can be transformed to a vertex-disjoint one using not more than 4 layers [5].

Manhattan routing is further classified in case of single row or channel routing problems. If each subgraph, corresponding to a net, contains a single horizontal wire segment only then the solution is called *dogleg-free.* Such a realization clearly minimizes the number of necessary vias. Using this observation the concept of dogleg-free solution may be defined for the gamma routing problem as well.

3 Results in 2-layer routing

1. Most of the literature concentrates on the **2-layer Manhattan model.**

Recall that the principal direction of a single row or a channel routing problem is *horizontal*, since the terminals are in the Northern (and possibly in the Southern) boundary. The *congestion* of a column (that is, a vertical straight line) is defined as the number of nets cut by this line into two. The *density* of a single row routing problem or a channel routing problem is the maximum congestion (taken over all columns). This quantity d is clearly a lower bound for the width of any 2-layer Manhattan solution.

Theorem 1. *A greedy interval packing algorithm solves the single row Manhattan routing problem in linear, that is, in $O(l)$ time. The result is of minimum width (satisfying $w = d$) and is dogleg-free.*

This basic result is probably due to Gallai [16]. Several authors (like [9, 47, 48]) attribute it to [22] under the name of *left edge algorithm,* and many described it again and again, without references, see [8] or [25], for example.

The slightly more complex task of determining the minimum *length* solution of the single row Manhattan routing problem turns out to be **NP**-hard [44], [45].

Channel routing (with or without doglegs) is an **NP**-hard problem in the Manhattan model [24], [46], see Remark 3 as well. However, there is a linear time algorithm [43] to decide if channel routing can be performed with width w in the Manhattan model. (The author supposes that the length l of the channel is part of the input but w is not. The number of operations is cl where the constant c depends on w in a superpolynomial way. A very similar philosophy appears in [14] for multilayer channel routing.)

While channel routing in our definition is **NP**-hard, it is always solvable if we may extend the length of the channel by introducing additional columns. In this case an algorithm of time complexity $O(lw)$ is given in [1]. They can achieve width at most $d+O(l^{1/2})$ for bipartite and at most $2d + O(l^{1/2})$ for the general problems (the coefficient 2 has later been reduced to 3/2, see [18, 19]. This time complexity is very good since due to the complicated shape of the wires the length of the output is also $O(lw)$.

Since channel routing is **NP**-hard, so are the usual and open switchbox routing problems as well.

Among the geometric patterns given in Chapter 1, gamma routing has not been considered yet. Its complexity in the Manhattan model is appearently unknown. We have a partial result:

Theorem 2. [4] *There is a linear time, that is, $O(l + w)$ algorithm to decide if a gamma-routing problem can be solved without doglegs in the Manhattan model, provided that every net consists of either at most one Northern (and an arbitrary number of Western) terminals or vice versa.*

Remark 3. Let us mention that, in comparison, the channel routing problem (with or without doglegs) remains **NP**-hard even if every net consists of either two Northern or two Southern terminals [20, 31].

An interesting consequence of the algorithm mentioned in Theorem 2 is that if the leftmost column or the top track of the routing area is free (that is, if the leftmost position of the Northern boundary or the top position of the Western boundary does not belong to any net) then the above gamma routing problem is always solvable [44].

Remark 4. We have seen at the end of Section 1 that single row and channel routing problems can be formulated both as decision and as minimization problems while the gamma and the (usual and open) switchbox routing problems are decision problems only. However, if additional tracks and columns may be introduced (without containing terminals) then these latters can also be considered as minimization problems. In this latter sense every switchbox routing problem can be solved in the Manhattan model, even in linear time [28], but not necessarily in minimum area.

2. If we consider **2-layer knock-knee routing,** the problem of determining whether the channel routing problem can be solved with a given width is still **NP**-complete [40]. In the single layer edge disjoint model one can always reach $w = d$ for the bipartite and $w = (3d/2) + O(d^{1/2})$ for the general problem [15, 18, 19] but the vertex disjoint realization of these solutions might require up to four layers, see Remark 2 above.

3. If we turn to the **unconstrained 2-layer model** then the complexity of determining the minimum width seems to be unknown even for the single row routing problem (however, see [34, 35, 38] for some partial results).

The existence of a solution for channel routing *in a given width* has been conjectured to be **NP**-complete [23]. But if one does not insist on a given width, or on minimum width, then, unlike in case of the Manhattan model, at least every channel routing problem can be solved [29, 39], even if we restrict ourselves to the so called *vertical unit-length overlap model* (two wires of different layers may run on top of each other for one vertical unit), see [2, 17, 18, 19]. If overlaps in both direction and in any length are permitted (the "real" unconstrained case) then even linear time is possible:

Theorem 3. [39] *Every bipartite channel routing problem can be wired with $O(d)$ width in the unconstrained 2-layer model. The time complexity of the algorithm is proportional to the area, that is, $O(ld)$.*

Theorem 4. [2, 17, 18, 19] *Every channel routing problem can be wired, even in the vertical unit-length overlap model, with width $(3d/2) + O(d^{1/2})$ (even $d + O(d^{1/2})$ for the bipartite case). The time complexity of the algorithm is $O(ld^{2/3})$.*

Theorem 5. [34, 38] *There is a linear time, that is, $O(l)$ algorithm to solve the channel routing problem in the unconstrained 2-layer model, without using doglegs.*

The theoretical upper bound for the resulting width in this last algorithm is very bad (although our limited numerical experience was promising), we obtained $w \le l$ for the bipartite and $w \le (3l/2)$ for the general case. Although the latter constant can be improved [M. Lengyel, unpublished], the real improvement would be an upper bound of form $w = O(d)$, still in linear time.

4 Results in multilayer routing

1. The advantages of the algorithms of Theorems 3, 4 and 5 appear simultaneously in the 3-layer Manhattan model.

Theorem 6. *Gallai's greedy interval packing algorithm solves any channel routing problem in the 3-layer Manhattan model in linear, that is, in $O(l)$ time with width $w = d$ and without doglegs.*

The very easy algorithm (pack the intervals, as in Theorem 1, into the second layer and perform every Northern interconnection in the first and every Southern interconnection in the third layer) belongs to the folklore (it dates back at least to [8]). Of course, the theoretical lower bound of the width is $\lceil d/2 \rceil$ since one may reserve the first and third layers for horizontal and the second layer for vertical wire segments rather than the other way round, see [11], for example. However, in this latter case, where $w = \lceil d/2 \rceil$ can trivially be achieved for the *single row* routing problem, the problem of minimum width *channel* routing is **NP**-hard, see [37], where an interesting relation between channel routing and multiprocessor scheduling, first discovered by [12], is also discussed. (The first relation of routing problems to scheduling dates back to [13], but the authors gave an LP-formulation only, appearently with no reference to the underlying combinatorial structure.)

2. In general, k-layer Manhattan model has two meanings for k odd: The VHVH...HV model (where more layers are reserved for vertical than for horizontal wire segments) and the HVHV...VH model (the other way round). A straightforward modification of Theorem 5 (see [6, 7, 14]) solves the former problem (every channel routing problem can be solved in $O(l)$ time without doglegs using $\lceil d/k_H \rceil$ tracks, where $k_H = (k-1)/2$ is the number of layers reserved for horizontal wire segments; and clearly this result is best possible). The latter problem appears to be difficult (although its complexity seems to be open for $k \geq 5$), see Remark 5.

If k is even then deciding the existence of a routing with width $\lceil d/k_H \rceil$ is known to be **NP**-hard [37], see also Remark 5.

Remark 5. If $k \geq 4$ is even or if the HVHV...VH model is considered for $k \geq 5$ then we have $k_H = \lfloor (k+1)/2 \rfloor$ layers reserved for horizontal wires, hence the lower bound $\lceil d/k_H \rceil \leq w$ is obvious and disregarding the external horizontal layer(-s) the algorithm of Theorem 6 always produces width $\lceil d/(k_H - 1) \rceil$ in linear time. The more complicated algorithms of [6, 7] produce, still in linear time, asymptotically better solutions, but they yield smaller width for very large d's only. On the other hand, the heuristics suggested in [37] usually gives better width, but its time complexity is slightly superpolynomial.

3. When we moved from single row routing to the more general channel routing problem, we had to permit unconstrained 2-layer technology rather than its special case, the Manhattan model. Analogously, one might expect that if we turn to the most general **multilayer model** then even the most general switchbox routing problem can be solved, see the table below.

This is only partially true: there is no absolute constant k so that every switchbox routing problem could be solved in k layers (see [3] but our idea was already implicitly contained in [21] some 10 years earlier). The reason of this negative result is that the width w and the length l of the switchbox may be very much different. Let $m = \max(l/w, w/l)$.

Theorem 7. [3] *There is a linear time, that is, $O(l+w)$ algorithm to solve the switchbox routing problem in k layers where k depends on the above quantity m only.*

This result was significantly improved [42]. While the algorithm in [3] required $2\lceil m \rceil + 14$ layers in the multilayer *unconstrained* model (or 18 layers if $m \leq 2$), his linear time algorithm requires $2\lceil m \rceil + 4$ layers only, moreover, he uses the multilayer *Manhattan* model. In particular, for square shaped switchboxes (with $m = 1$) his algorithm needs 6 layers. An example in [3] shows that at least 4 layers are necessary for certain specifications.

	2-layer Manhattan model	2-layer unconstrained model	3-layer Manhattan (VHV) model	3-layer Manhattan (HVH) model	....	multilayer Manhattan model
single row routing	Theorem 1. $w = d$ **and** in linear time			$w = \lceil d/2 \rceil$ **and** in linear time		
channel routing	known to be **NP**-hard	Theorems 3-4. $w = O(d)$ **or** Theorem 5. (linear time)	Theorem 6. $w = d$ **and** in linear time	known to be **NP**-hard		
switchbox routing	known to be **NP**-hard					Theorem 7. (linear time)

5 Some open problems

- Find an $O(l)$ time algorithm which solves the channel routing problem with width $O(d)$ in the unconstrained 2-layer model.
- Is every channel routing problem solvable with $w < d$ in the unconstrained 3-layer model (if $d \geq 4$)?
- Is every single row routing problem solvable with $w < \lceil d/2 \rceil$ in the unconstrained 3-layer model (if $d \geq 5$)?
- Is the switchbox routing problem solvable in the unconstrained multilayer model using $k = (2-\varepsilon)m + c$ layers for some constant c?
- What is the minimum number p of layers necessary to wire every square shaped switchbox? We saw that $p \geq 4$ even in the unconstrained model and $p \leq 6$ even in the Manhattan model.
- Determine the time complexity of the minimum width channel routing in the HVHV...VH model (for $k \geq 5$ odd).

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