Theory of Algorithms Midterm, 29 March 2007

Write your name and a code of at least 5 characters, in print, onto each sheet you turn in. The code will be used to put up the results on the web page http://www.cs.bme.hu/~pbiro/alg.html preserving your privacy.

- 1. Let $f(n) = 15n^2 30n + 45 \log n$. Which statements are true? Explain why.
 - (a) $f(n) = O(n^3)$
 - (b) $f(n) = \Theta(n^2)$
 - (c) $f(n) = \Omega(n)$
- 2. Give the running time of INSERTION SORT, MERGE SORT, QUICKSORT, HEAPSORT and the expected running time of RANDOMIZED-QUICKSORT using Θ -notation, if the input array is
 - (a) already sorted,
 - (b) in reverse order.
- 3. Illustrate the operations of QUICKSORT on the array A = [2, 8, 10, 7, 1, 3].
- 4. The operation HEAP-DELETE(A,i) deletes the item in node i from heap A. Give the implementation of HEAP-DELETE that runs in $O(\log n)$ time for an n-element max-heap.
- 5. The array A of size n was already ordered, then somebody switched its elements, but not dramatically: the distance between the new and the original positions is at most 5 for each of the elements. Give an O(n) time algorithm that reorders the array.
- 6. What is the worst case running time for the BUCKET-SORT algorithm? What simple change to the algorithm preserves its linear expected running time and makes its worst-case running time $O(n \log n)$?

Each problem is worth 5 points, you need 12 points at least to pass. Your point total in the final exam counts for 30% of your final grade.