Theory of Algorithms Final December 22, 2006

Write your name and a code of at least 5 characters, in print, onto each sheet you turn in. The code will be used to put up the results on the web page `http://www.cs.bme.hu/~pbiro/thalg.html` preserving your privacy.

1. What is a heap, how do we build a heap, what is the time necessary to build a heap? How can heaps be used for priority queues?

2. Give an algorithm that for an input of an array $A[1 \dots n]$ of n numbers returns the 117th largest element of A in $O(n)$ time.

3. Show how quicksort can be made to run in $O(n \lg n)$ time in the worst case.

4. Show how to implement a stack using two queues. Analyze the running time of the stack operations.

5. We inserted $2n$ elements into the open address hash-table $T[0...M-1]$ and they went to the first $3n$ slots ($M - 1 > 3n$) with the help of an unknown hash-function. Every slot of index $3i$ ($0 \leq i < n$) remained empty. How many collisions could there be at most if we used linear probing for collision resolution?

6. Consider a binary search tree T whose keys are distinct. Show that if the right subtree of a node x in T is empty and x has a successor y, then y is the lowest ancestor of x whose left child is also an ancestor of x. (Recall that every node is its own ancestor.)

7. There are two types of professional wrestlers: good guys and bad guys. Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n + r)$-time algorithm that determines whether it is possible to designate some of the wrestlers as good guys and the remainder as bad guys such that each rivalry is between a good guy and a bad guy. If is it possible to perform such a designation, your algorithm should produce it.

Each problem is worth 10 points, you need 28 points at least to pass. Your point total in the final exam counts for 70% of your final grade.