Theory of Algorithms Final January 11, 2007

Write your name and a code of at least 5 characters, in print, onto each sheet you turn in. The code will be used to put up the results on the web page `http://www.cs.bme.hu/~pbiro/thalg.html` preserving your privacy.

1. Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $32n \lg n$ steps. For which values of n does insertion sort beat merge sort?

2. What is a balanced partitioning? Show that the total cost of quicksort is $O(n \lg n)$ if the partitioning algorithm always produces a 9-to-1 proportional split.

3. We have given a MAX-HEAP of size $2^n - 1$. How many comparisions do we exactly need if we have to find the SMALLEST element?

4. Suppose that we have numbers between 1 and 1000 in a binary search tree and want to search for the number 363. Which of the following sequences could not be the sequence of nodes examined?

   (a) $2, 252, 401, 398, 330, 344, 397, 363$.

   (b) $925, 202, 911, 240, 912, 245, 363$.

5. We have given n points in the plain with their integer coordinates: $P_1(x_1, y_1), P_2(x_2, y_2), \ldots, P_n(x_n, y_n)$. Give an $O(n \log n)$ time algorithm to decide whether there exist two points P_i, P_j $(i \neq j)$, such that their distance is at most 2.

6. Implement a queue by a singly linked list L. The operations ENQUEUE and DEQUEUE should still take $O(1)$ time.

7. Given a directed graph $G(V, E)$. Show that there exists a E_1, E_2 partition of the edges ($E = E_1 \cup E_2$ and $\emptyset = E_1 \cap E_2$), where neither of the subgraphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ has a directed cycle. Give an algorithm to find such a partition.

Each problem is worth 10 points, you need 28 points at least to pass. Your point total in the final exam counts for 70% of your final grade.