Theory of Algorithms Final January 4, 2007

Write your name and a code of at least 5 characters, in print, onto each sheet you turn in. The code will be used to put up the results on the web page `http://www.cs.bme.hu/~pbiro/thalg.html` preserving your privacy.

1. A software solves a problem of size k in a day if we run the program on our old computer. We bought a new computer that is 100-times faster than the old one. What is the approximate size of a problem that we can solve on the new computer in a day, supposing that for an input of size n the running time of our software is

   (a) $\Theta(n^3)$

   (b) $\Theta(2^n)$

2. Analyse the running time of the merge sort algorithm.

3. Illustrate the operation of HEAPSORT on the array $< 5, 1, 10, 6, 2, 4 >$.

4. How many comparisons do we exactly need if we have to find one of the 10 smallest elements from an unsorted array of size n?

5. We have given a complete binary search tree of size $n = 2^k - 1$. The keys are different integers from the interval $I = [1, 2^k]$. This implies that only one single integer from I is *not* presented in the tree. Give an $O(k)$-time procedure to determine that integer.

6. Is the hash function $h(K) = K^2 \pmod 7$ is good if the table size is 7?

7. Given a directed graph $G(V, E)$ with weights 1, 2 or 3 on the edges. Give an $O(|V| + |E|)$-time algorithm that finds the minimum-weight paths from a given node s to each of the other nodes.

Each problem is worth 10 points, you need 28 points at least to pass. Your point total in the final exam counts for 70% of your final grade.