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The stable matching problem and its generalizations: an algorithmic and game theoretical approach

We say that a market situation is stable in a general sense, if there is no set of agents such that all of them are interested in creating a new cooperation (and breaking their other eventual cooperations). Gale and Shapley [7] introduced and studied the problem of stable marriages as a special question. There, a matching, that corresponds to a set of marriages, is stable, if there exists no man and woman that would both prefer to marry each other (after leaving their eventual partners). Gale and Shapley described a natural algorithm that finds a stable matching for the marriage problem, so when the underlying graph is bipartite.

In the first chapter, we give a general overview on Cooperative Game Theory and we define stable matchings as core elements of a certain NTU-game.

The nonbipartite stable matching problem, the so-called stable roommates problem is the subject of Chapter 2. We study the dynamics of the stable marriages and stable roommates solutions. We lean on the algorithm of Roth and Vande Vate [8] in the bipartite case, and on a similar algorithm of Tan and Hsueh [9] in the one-sided case to model the dynamics of the matching market. We analyze the properties of the solutions obtained by these incremental algorithms (see [5]). We also study the complexity of the problem of "almost stable matchings", that is the problem of finding a matching for a roommates problem with the fewest number of blocking pairs (see [1]).

In Chapter 3, we focus on stable matching problems with vertex-bounds and edge-capacities. We recall Scarf's lemma and show that if all the bounds and capacities are integers, then the so-called integral stable allocation problem for graphs can be reduced to the stable roommates problem by a sequence of constructions. We describe a strongly polynomial algorithm created by Baïou and Balinski for two-sided matching markets, and we study its generalization in the one-sided case. Finally, as a practical application of stable b-matchings, we describe the higher education admission program in Hungary (see [2]).

The last chapter is devoted to the question of exchanging indivisible goods. First we give a game theoretical overview and we describe the motivation: the kidney exchange problem. Afterwards, we consider the maximum weight exchange problem with restrictions (see [6]). Finally, we study the so-called stable exchange problems (see [3]) and we present also a result of [4] about the inapproximability of a specific stable exchange problem.

References

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