Solution for Home-work of last time: Let X be a random variable with density function

$$f(x) = \begin{cases} \frac{3x^2}{8}, & \text{if } x \in [0,2] \\ 0, & \text{otherwise.} \end{cases}$$

The source is quantized by a 2-level quantizer. Starting from the initial levels $\frac{1}{2}$ and $\frac{3}{2}$, give the first iteration (first two steps) of the Lloyd-Max algorithm.

$B_1 = [0, y_1]$ $\quad B_2 = (y_1, 2]$ $\quad y_1 = \frac{1/2 + 3/2}{2} = 1$

$x_2 = E(X \mid X \in B_2)$

$$x_1 = E(X \mid X \in B_1) = \frac{\int_{B_1} x f_X(x)\,dx}{\int_{B_1} f_X(x)\,dx} = \frac{\int_0^1 x \frac{3x^2}{8}\,dx}{\int_0^1 \frac{3x^2}{8}\,dx} = \frac{\left[\frac{3}{8}\frac{x^4}{4}\right]_0^1}{\left[\frac{x^3}{8}\right]_0^1} = \frac{3/32}{1/8} = \frac{3}{4}$$

$$x_2 = \frac{\int_{B_2} x f_X(x)\,dx}{\int_{B_2} f_X(x)\,dx} = \frac{\int_1^2 x \frac{3x^2}{8}\,dx}{\int_1^2 \frac{3x^2}{8}\,dx} = \frac{\left[\frac{3}{8}\frac{x^4}{4}\right]_1^2}{\left[\frac{x^3}{8}\right]_1^2} = \frac{\frac{3}{8}\cdot\frac{16}{4} - \frac{3}{8}\cdot\frac{1}{4}}{\frac{8}{8} - \frac{1}{8}} = \frac{3/2 - 3/32}{7/8} = \frac{\frac{48-3}{32}}{7/8}$$

$$= \frac{45/32}{7/8} = \frac{45}{7 \cdot 4} = \frac{45}{28}$$

Home-work 1:

(One bit quantization of a single Gaussian random variable)

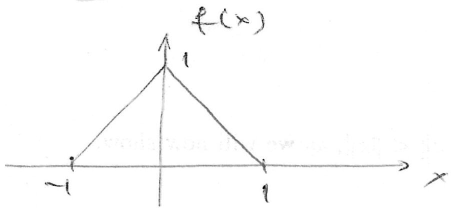
Let $X \sim N(0, \sigma^2)$ r.v.

Find the optimal 1 bit quantizer!
(2 level)

Distortion = ?

Home-work 2:

Quantize X with a 2 bit uniform quantizer



a) Calculate the exact distortion and entropy of the quantizer

b) Calculate them by using the appr.