Information Theory—Repeated midterm test, 25 November 1999

Important! Answers are not complete without sufficient reasoning.

Problem 1 *(a) Consider a fair coin flip. What is the mutual information between the top side and the bottom side of the coin?*
(b) A 6-sided fair dice is rolled. What is the mutual information between the top side and the front face (the side most facing you)?

Problem 2 *What is the maximum entropy probability mass function $p(x, y)$ with the following marginals?*

	x_1	x_2	x_3	
y_1	p_{11}	p_{12}	p_{13}	*1/2*
y_2	p_{21}	p_{22}	p_{23}	*1/4*
y_3	p_{31}	p_{32}	p_{33}	*1/4*
	2/3	*1/6*	*1/6*	

Find $H(X, Y)$ for the above distribution.

Problem 3 *Find an optimal* (a) binary *and* (b) ternary *prefix code for the random variable X with distribution $(1/21, 2/21, 3/21, 4/21, 5/21, 6/21)$. Calculate the expected codelength in each case.*

Problem 4 *Let $\mathbf{Z} = Z_1, Z_2, \ldots$ be a binary, stationary Markov chain with transition probabilities*

$$\mathbf{P}\{Z_2 = 0|Z_1 = 0\} = \frac{3}{4},\ \mathbf{P}\{Z_2 = 1|Z_1 = 0\} = \frac{1}{4},\ \mathbf{P}\{Z_2 = 0|Z_1 = 1\} = \frac{1}{4},\ \mathbf{P}\{Z_2 = 1|Z_1 = 1\} = \frac{3}{4}.$$

Find the stationary distribution and the entropy of the chain.
Construct a Huffmann code to encode blocks of length 3. Find the expected codelength per symbol.

Problem 5 *Give the definition of the Markov-source. How can its entropy rate be calculated?*