Information Theory—Midterm test, 22 November 2001

Important! Answers are not complete without sufficient reasoning.

Problem 1 *Assume that the source alphabet is binary. Is it possible to construct a uniquely decodable code that consists of* 8 *codewords and the lengths of the codewords are* $2, 2, 3, 3, 3, 5, 5, 5$*? Why?*

Problem 2 *A biased coin* ($\mathbf{P}(head) = \frac{1}{3}, \mathbf{P}(tail) = \frac{2}{3}$) *is tossed until two heads or two tails are seen. Let* X_1 *and* X_2 *be the outcomes of the first two tosses,* Y *the outcome of the last (second or third) toss and* N *the number of tosses. Calculate* $H(X_1)$, $H(X_2)$, $H(Y)$, $H(N)$, $I(X_1, Y)$, $I(X_2, Y)$, $I(X_1, X_2; Y)$, $I(Y, N)$, $I(X_1, N)$, $I(X_2, N)$, $I(X_1, X_2; N)$.

Problem 3 *Let* $X_1, \ldots, X_n$ *be binary random variables. Denote by* $\mathbf{R} = (R_1, R_2, \ldots)$ *the run lengths of this sequence. For example the sequence* 1110010001111 *yields run lengths* $\mathbf{R} = (3, 2, 1, 3, 4)$*. Compare* $H(X_1, \ldots, X_n), H(\mathbf{R})$ *and* $H(\mathbf{R}, X_n)$*. Show all equalities and inequalities.*

Problem 4 *Find a possible Huffman code and the Lempel-Ziv-Welch code of the word MISSISSIPPI. Concerning the latter, assume that the letters I,M,P and S are already in the dictionary and their codes are 1,2,3 and 4 respectively. Describe also the final state of the dictionary (words and their codes).*

Problem 5 *Give the definition of the Markov-source. How can its entropy rate be calculated?*

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Problem 6 *Assume that the source alphabet is binary. Is it possible to construct a uniquely decodable code that consists of* 8 *codewords and the lengths of the codewords are* $2, 2, 3, 3, 3, 5, 5, 5$*? Why?*

Problem 7 *A biased coin* ($\mathbf{P}(head) = \frac{1}{3}, \mathbf{P}(tail) = \frac{2}{3}$) *is tossed until two heads or two tails are seen. Let* X_1 *and* X_2 *be the outcomes of the first two tosses,* Y *the outcome of the last (second or third) toss and* N *the number of tosses. Calculate* $H(X_1)$, $H(X_2)$, $H(Y)$, $H(N)$, $I(X_1, Y)$, $I(X_2, Y)$, $I(X_1, X_2; Y)$, $I(Y, N)$, $I(X_1, N)$, $I(X_2, N)$, $I(X_1, X_2; N)$.

Problem 8 *Let* $X_1, \ldots, X_n$ *be binary random variables. Denote by* $\mathbf{R} = (R_1, R_2, \ldots)$ *the run lengths of this sequence. For example the sequence* 1110010001111 *yields run lengths* $\mathbf{R} = (3, 2, 1, 3, 4)$*. Compare* $H(X_1, \ldots, X_n), H(\mathbf{R})$ *and* $H(\mathbf{R}, X_n)$*. Show all equalities and inequalities.*

Problem 9 *Find a possible Huffman code and the Lempel-Ziv-Welch code of the word MISSISSIPPI. Concerning the latter, assume that the letters I,M,P and S are already in the dictionary and their codes are 1,2,3 and 4 respectively. Describe also the final state of the dictionary (words and their codes).*

Problem 10 *Give the definition of the Markov-source. How can its entropy rate be calculated?*