

## Information Theory—Repeated midterm test, 7 December 2000

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**Important!** Answers are not complete without sufficient reasoning.

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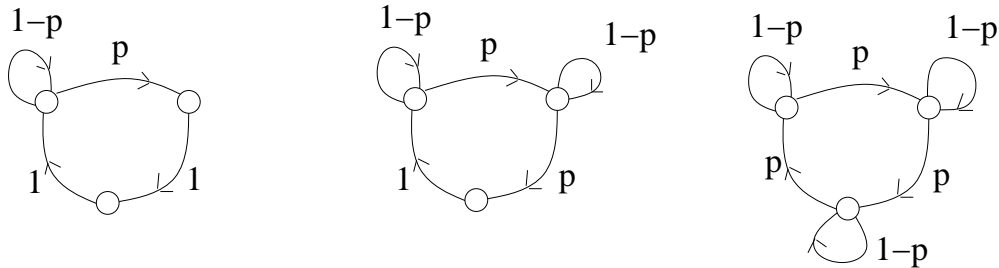
**Problem 1** Let  $X$  be a discrete random variable. Compare (establish inequalities or equalities)  $H(X)$  and  $H(Y)$ , where (a)  $Y = 2^X$ . (b)  $Y = \cos X$ .

**Problem 2** (a) Consider a fair coin flip. What is the mutual information between the top side and the bottom side of the coin?

(b) A 6-sided fair dice is rolled. What is the mutual information between the top side and the front face (the side most facing you)?

**Problem 3** The outcome of an experiment is a random variable with seven possible values and probability distribution  $(1/3, 1/3, 1/9, 1/9, 1/27, 1/27, 1/27)$ . We want to transmit the result. We can transmit it either in binary form for 200 Ft per bit or in ternary form for 325 Ft per ternary symbol. Which one should we choose and what code should we use to minimize the expected cost? Which one should we choose if the experiment is repeated many times independently and we can code the results jointly? ( $\log 3 \approx 1.585$ )

**Problem 4** Find the entropy rate of the following three stationary Markov chains. Establish inequalities or equalities between the entropies.



**Problem 5** Give the precise definition of uniquely decodable codes. What is the difference between McMillan's and Kraft's inequality?