Information Theory—Exam paper, 22 December 1999

Important! Answers are not complete without sufficient reasoning.

Problem 1 *Let X be a random variable with exponantial distribution ($f(x) = \lambda e^{-\lambda x}$). Quantizing X uniformly, the entropy of the output of the quantizer is 16 bits. Determine approximately the singal-noise ratio in dB ($10 \log_{10} \frac{\mathbf{E}X^2}{D(Q)}$).*

Problem 2 *A dog walks on the integers, possibly reversing direction at each step with probability 0.1. Let $X_0 = 0$. The first step is equally likely to be positive or negative. A typical walk might look like this:*

$$(X_0, X_1, \ldots) = (0, 1, 2, 3, 4, 5, 4, 3, 2, 1, 0, -1, \ldots)$$

Find $H(X_1, X_2, \ldots, X_n)$, find the entropy rate of this browsing dog.
What is the expected number of steps the dog takes before reversing direction?

Problem 3 *Let $l_1, l_2, \ldots, l_{10}$ be the binary Huffman codeword lengths for the probabilities $p_1 \geq \ldots \geq p_{10}$. Suppose we get a new distribution by splitting the last probability mass. What can you say about the optimal binary codeword lengths $\tilde{l}_1, \tilde{l}_2, \ldots, \tilde{l}_{11}$ for the probabilities $p_1, p_2, \ldots, p_9, \alpha p_{10}, (1-\alpha)p_{10}$, where $0 \leq \alpha \leq 1$. (why?)*

Problem 4 *Show that the capacity of BSC(p) (binary symmetric channel) is $1 - h(p)$.*

Problem 5 *Give a definition of source coding with fidelity criterion. Define the rate distortion function, and state the rate distortion theorem and its converse.*