Information Theory—Exam paper, 10 January 2003

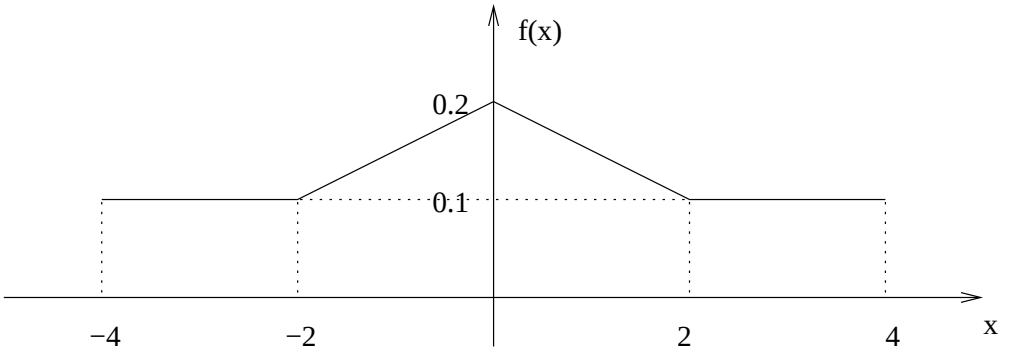
Important! Answers are not complete without sufficient reasoning.

Problem 1 *Give the definition of Markov source. How can its entropy rate be calculated?*

Problem 2 *Let the source alphabet be* $\mathcal{X} = \{a, b, c\}$ *and the initial dictionary contain the letters* a, b *and* c *with their codewords* $(1, 2$ *and* 3 *respectively). Using the Lempel-Ziv-Welch algorithm,*
(a) encode the sequence **cabcbcbcb**,
(b) decode the sequence $\mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}, \mathbf{7}, \mathbf{1}$.

Problem 3 *Define the rate-distortion function, and state the rate-distortion theorem and its converse.*

Problem 4 *We quantize the random variable* X *with a two bit uniform quantizer fitted to the interval* $[-4, 4]$. *The density function of* X *is shown in the figure.*
(a) Find the squared distortion of the quantizer, both the exact and the estimated values.
(b) Find the entropy of the quantizer.



Problem 5 *Find the capacity of the channel given by the following figure (binary Z channel with parameter* $1/2$*).*