Information Theory—Exam paper, 17 January 2002

Important! Answers are not complete without sufficient reasoning.

Problem 1 *What is the maximum entropy probability mass function* $p(x,y)$ *with the following marginals?*

	x_1	x_2	x_3	
y_1	p_{11}	p_{12}	p_{13}	*1/2*
y_2	p_{21}	p_{22}	p_{23}	*1/4*
y_3	p_{31}	p_{32}	p_{33}	*1/4*
	2/3	*1/6*	*1/6*	

Find $H(X,Y)$ *for the above distribution.*

Problem 2 *State the source coding theorem, lower and upper bounds for the expected codeword length.*

Problem 3 *Let* $l_1, l_2, \ldots, l_{10}$ *be the binary Huffman codeword lengths for the probabilities* $p_1 \geq \ldots \geq p_{10}$. *Suppose we get a new distribution by splitting the last probability mass. What can you say about the optimal binary codeword lengths* $\tilde{l}_1, \tilde{l}_2, \ldots, \tilde{l}_{11}$ *for the probabilities* $p_1, p_2, \ldots, p_9, \alpha p_{10}, (1-\alpha)p_{10}$, *where* $0 \leq \alpha \leq 1$. *(why?)*

Problem 4 *Let* X *be uniformly distributed over the interval* $[0, 50]$. *Quantizing* X *uniformly, the distortion is* 0.02. *Give a good estimation for the length of the quantization regions* (q). *Find the entropy of the quantizer.*

Problem 5 *Define the channel capacity, give the capacity of BSC(p) (binary symmetric channel) with proof..*