Information Theory—Exam paper, 25 January 2001

Important! Answers are not complete without sufficient reasoning.

Problem 1 *A playoff consists of a three-game series that terminates as soon as either team wins two games. Let X be the random variable that represents the outcome of a playoff between teams A and B; examples of possible values of X are AA, BAB etc. Let Y be the number of games played, which ranges from 2 to 3.*

   (a) Assuming that A and B are equally matched and that the games are independent, calculate $H(X)$, $H(Y)$, $H(Y|X)$ and $H(X|Y)$.

   (b) Let Z denote the winning team. Find $H(X|Z)$. Compare to $H(X)$. Find $H(Z|X)$.

   (c) Find $I(Y;Z)$.

Problem 2 *Find a possible dynamic Huffman code and the Lempel-Ziv-Welch code of the word MISSISSIPPI. Concerning the latter, assume that the letters I,M,P and S are already in the dictionary and their codes are 1,2,3 and 4 respectively. Describe also the final state of the dictionary (words and their codes).*

Problem 3 *Find the uniform 2 bit quantizer of the random variable X uniformly distributed over the interval $[0, 8]$. Calculate exactly the distortion and the entropy of the quantizer. Find the differential entropy. Using the estimates discussed in class, give approximate values for the distortion and the entropy of the quantizer. Compare the esimates to the exact values.*

Problem 4 *Show that the capacity of BSC(p) (binary symmetric channel) is $1 - h(p)$.*

Problem 5 *State and prove Jensen's inequality.*