Information Theory—Exam paper, 20 January 2000

Important! Answers are not complete without sufficient reasoning.

Problem 1 *Let X be a random variable with density function $f(x) = \frac{1}{2}\lambda e^{-\lambda|x|}$. Quantizing X uniformly, the entropy of the output of the quantizer is 16 bits. Find the differential entropy of f. Using the approximations learned, determine the distortion of the quantizer.*

Problem 2 *A fair coin is tossed until two heads or two tails are seen. Let X_1 and X_2 be the outcomes of the first two tosses, Y the outcome of the last (second or third) toss and N the number of tosses. Calculate $H(X_1)$, $H(X_2)$, $H(Y)$, $H(N)$, $I(X_1, Y)$, $I(X_2, Y)$, $I(X_1, X_2; Y)$, $I(Y, N)$, $I(X_1, N)$, $I(X_2, N)$, $I(X_1, X_2; N)$.*

Problem 3 *Let $\mathbf{X} = (X_1, X_2, \ldots)$ be a stationary source with entropy rate $H(\mathbf{X})$. Calculate the entropy rate of the following sources (if the entropy rate exists):*

a) $\mathbf{X}_a = (X_1, X_1, X_2, X_2, X_3, X_3, \ldots)$ *(i.e. we repeat every random variable)*

b) $\mathbf{X}_b = (X_1, X_1, X_2, X_3, X_3, X_4, X_5, X_5, X_6, \ldots)$ *(i.e. we repeat the random variables with odd index)*

c) $\mathbf{X}_c = (X_1, X_2, X_2, X_3, X_3, X_3, X_4, X_4, X_4, X_4, \ldots)$ *(i.e. we repeat the ith random variable i times)*

Problem 4 *Show that the capacity of the binary erasure channel is $1 - p$.*

Problem 5 *Give a definition of source coding with fidelity criterion. Define the rate distortion function, and state the rate distortion theorem and its converse.*