Combinatorial optimization

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1. Solve these linear programs by the graphical method.

$$\begin{aligned} x &\geq 1 \\ 2x + y &\leq 4 \\ x - 3y &\leq 9 \\ \min\ & x + 2y \end{aligned} \qquad \begin{aligned} 2x - y &\leq 8 \\ -5x - 2y &\leq 8 \\ -4x + 6y &\leq 24 \\ \max\ & 2x + 6y \end{aligned} \qquad \begin{aligned} 2y - x &\leq 4 \\ 4x + 6y &\geq 24 \\ 3x - 2y &\leq 12 \\ y &\geq 2 \\ \min\ & y - 2x \end{aligned}$$

2. A metallurgist has received an order for an iron rod made of a special alloy. The weight of the rod must be 2 tons and the alloy must contain at least 2.5% carbon and 2.2% silicon, and no more than 3.2% carbon and 2.8% silicon. The materials to be included and their specifications are shown in the following table.

Material	Cost/Ton ($)	Carbon %	Silicon %
Pig iron	30	5	2
Steel	60	0	0.5
Ferrosilicon	50	2	15

   The metallurgist is interested in, that what should be the quantities of the materials to be included in the charge so that the alloy can be produced at the cheapest cost.

   Help him. Formalize this problem as a linear program. You do not have to solve it.

3. A company supplies feed for chickens by mixing corn and wheat. Corn costs 1.1cents an ounce and wheat costs 1 cent an ounce. Each ounce of corn has 10 units of vitamin A, 5 calories and 2 units of protein. Each ounce of wheat has 4 units of vitamin A, 5 calories and 6 units of protein. Minimum daily requirements for a chicken are 20 units of vitamin A, 20 calories and 12 units of protein. What is the least cost mix of feed?

4. We know that $x = 70, y = 130$ is an optimal solution of the linear program given on the right.
   Determine the value of p.

$$\begin{aligned} x + y &\leq 200 \\ x - y &\leq 100 \\ y - x &\leq 100 \\ x &\geq 0 \\ y &\geq 0 \\ \max\ & px + 42y \end{aligned}$$

5. A contractor has five locations A, B, C, D, and E on a road contract to concrete is to be delivered. The concrete, which is all of the same quality, is to be supplied from concrete plants 1, 2, and 3. The table below shows the relative costs per tons of transporting concrete from each source to each location, the quantity which will be available at each plant and the quantities of concrete required at each location.

Concrete plant	Relative transportation costs (t)					Plant output
	A	B	C	D	E	(t)
1	5	10	12	11	10	120
2	12	8	6	3	7	205
3	2	9	4	13	6	150
Quantity required at each location (t)	55	105	150	135	90	

The contractor wants to allocate quantities of concrete from each plant to each location for the lowest total transportation cost.

Formalize this problem as a linear program. You do not have to solve the obtained LP or IP.

6. A canning company operates two canning plants. The growers are willing to supply fresh fruits in the following amounts:

Grower 1: 200 tonnes at 11€/tonne
Grower 2: 310 tonnes at 10€/tonne
Grower 3: 420 tonnes at 9€/tonne

Shipping costs in € per tonne are:

	To: Plant A	To: Plant B
From: Grower 1	3	3.5
From: Grower 2	2	2.5
From: Grower 3	6	4

The capacities of Plant A and Plant B are 400 tonnes and 500 tonnes, respectively. The cost of manufacturing is 35€/tonne and 30€/tonne in Plant A and Plant B, respectively. The canned fruits are sold at 50€/tonne to the distributors. The company can sell at this price all they can produce. The objective is to find the best mixture of the quantities supplied by the three growers to the two plants so that the company maximizes its profits.

Formalize this problem as a linear program. (Define variables, give constraints and the objective function.) You do not have to solve it.

7. Solve the following linear programming problem by the graphical method.

$$\begin{aligned} 4x - y &\geq 2 \\ 3x - 2y &\leq 6 \\ x + 6y &\leq 30 \\ y &\geq -1 \\ \max\ & x - 2y \end{aligned}$$

Is the vector $[2, 2]$ an element of the set of feasible solutions?