# Combinatorial optimization

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1. In the BIN PACKING problem we have 4 items. The sizes are $0.4, 0.7, 0.1$ and $0.6$.
   (a) Run the First Fit algorithm!
   (b) The First Fit Decreasing algorithm firstly sorts the items into descending order, then runs the Firs Fit algorithm for that new order. Run this First Fit Decreasing algorithm!
   (c) How many bins are utilized by the optimal packing?

2. HAMILTONIAN PATH is the following decision problem:

   Input: A graph $G$

   Question: Is there a Hamiltonian path in $G$?

   (a) Show that HAMILTONIAN PATH is in NP.
   (b) Show that a HAMILTONIAN PATH$\prec$ 3-SAT Karp reduction exists.
   (c) Give a Karp reduction from HAMILTONIAN (CYCLE) to HAMILTONIAN PATH.
   (d) Show that HAMILTONIAN PATH is an NP-complete problem.

3. Consider the Bin packing problem where the sizes of the items are the following: 0.15, 0.4, 0.25, 0.55, 0.55, 0.55, 0.55, 0.55, 0.2, 0.1, 0.1.
   (a) Run the First Fit algorithm. Is the result of this algorithm is an optimal packing?
   (b) Run the First Fit Decreasing algorithm. Is the the result of this algorithm is an optimal packing?

4. Let SHORT PATH be the following decision problem:

   Input: A graph $G$, vertices $u$, $v$ and a number $k$.

   Question: Is there a path between $u$ and $v$ whose length is at most $k$ (contains at most $k$ edges)?

   Assume that P$\neq$ NP. Under this assumption, do these Karp reductions exists?

   (a) SHORT PATH $\prec$ 3-SAT.
   (b) 3-SAT $\prec$ SHORT PATH
   (c) BIN PACKING $\prec$ HAMILTONIAN (CYCLE)

5. S-T HAMILTONIAN PATH is the following decision problem:

   Input: A graph $G$ and two vertices of $G$: $S$ and $T$.

   Question: Does $G$ contain a Hamiltonian path which starts with $S$ and ends with $T$?

   (a) Show that S-T HAMILTONIAN PATH is in NP.
   (b) Give an S-T HAMILTONIAN PATH $\prec$ HAMILTONIAN (CYCLE) Karp reduction.

6. Let BIPARTITE PERFECT MATCHING be the following decision problem:

   Input: A bipartite graph $G$.

   Question: Does $G$ have a perfect matching?

   Assume that $P \neq NP$. Under this assumption, do these Karp reductions exist?

   (a) 3-SAT $\prec$ BIPARTITE PERFECT MATCHING

   (b) BIPARTITE PERFECT MATCHING $\prec$ CLIQUE

7. We have the following input of the bin packing problem: 0.3, p, 0.6, 0.4, 0.3, q, 0.2, 0.15, 0.3. We ran the first fit algorithm and this is the obtained output:

   1st bin: 0.3, 0.6,

   2nd bin: p, 0.2,

   3rd bin: 0.4, 0.3, q,

   4th bin: 0.15, 0.3.

   Determine all the possible values of the (p,q) pair.

8. Let LONG PATH be the following decision problem:

   Input: A simple graph $G$ and a number $k$.

   Question: Is there a path in $G$ whose length is at least $k$ (contains at least $k$ edges)?

   (a) Show that LONG PATH is in NP.

   (b) Show that the LONG PATH $\prec$ 3-SAT Karp reduction exists.

9. Give a HAMILTONIAN PATH$\prec$ HAMILTONIAN (CYCLE) Karp reduction.

10. In all of these problems the input is a simple undirected graph $G$ and a set $S$ which is a subset of $V(G)$. Decide which ones of these problems are contained in P and which ones are NP-Complete?

    (a) Does $G$ contain a spanning tree $T$ where each element of $S$ is a leaf(a vertex is a leaf if its degree is one)?

    (b) Does $G$ contain a spanning tree $T$ whose leaf vertices are exactly the elements of $S$?

    (c) Does $G$ contain a spanning tree $T$ whose leaf vertices are contained in $S$?

11. Assume that we have an algorithm $A$ which decides the HAMILTONIAN problem in polynomial time. So it tells for each graph whether it contains a hamiltonian cycle or not. Design a polynomial time algorithm which uses $A$ several times and finds a hamiltonian cycle in any given graph.