Combinatorial optimization

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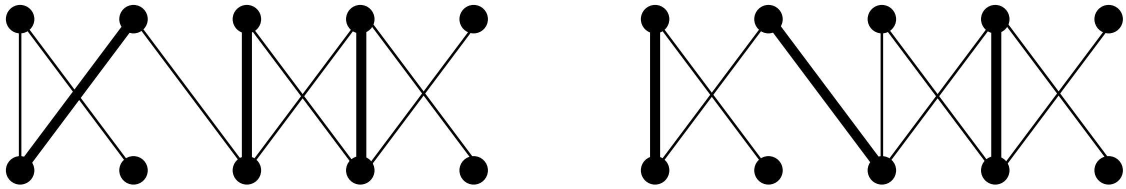
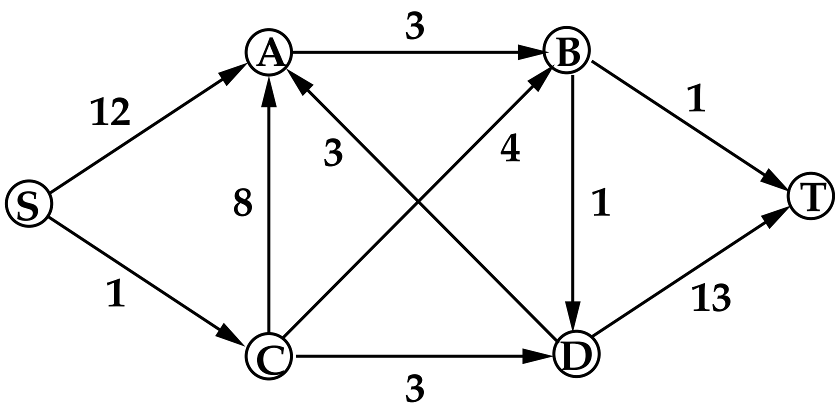
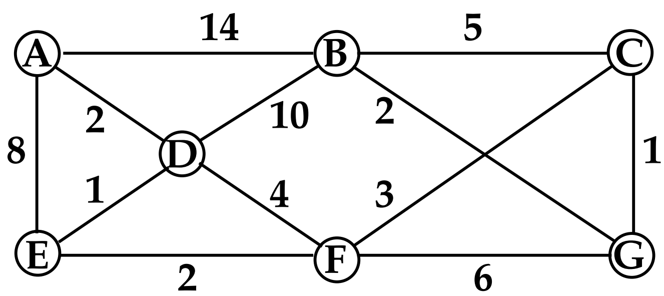
1. Are these matchings maximal? Are these matchings maximum? If none of them are maximum, then find a maximum matching by the algorithm which we have learnt to solve this problem.



2. Find the distance between S and every other vertex by running Dijkstra's algorithm. Give a shortest path between S and T.



3. Determine the distance between A and every other vertex. Also give a shortest path between A and B.

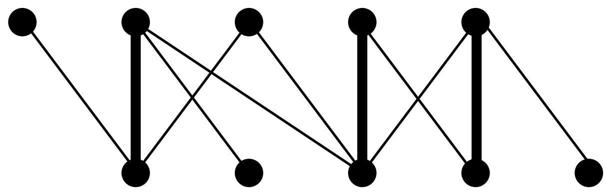


4. Let G be a simple graph and l be a non-negative length function over the edge set ($l : E(G) \to R^{+}$). Denote three different vertices of G by u,v, and w. Are these statements true or false?

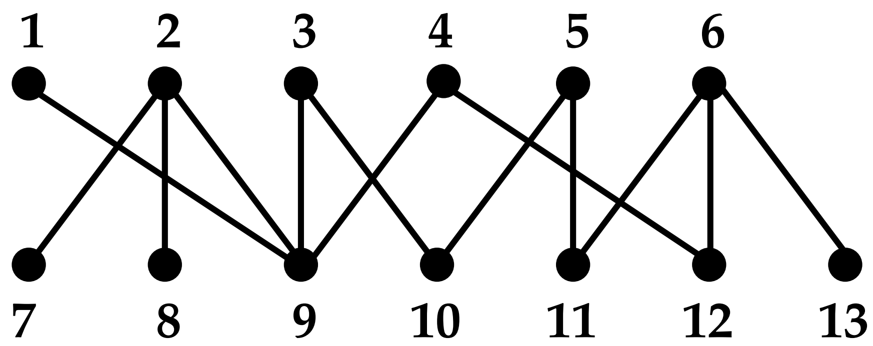
   (a) If P is a shortest path between u and v and it contains w, then its section between u and w is a shortest path between u and w.

   (b) If P_1 is a shortest path between u and w and similarly P_2 is a shortest path between w and v, then the concatenation of P_1 and P_2 (gluing together P_1 and P_2 at w) is a shortest path between u and v.

5. Is this matching maximal? Is it maximum? If it is not a maximum matching, then find one.



6. Consider the following graph.



   (a) Find a maximum matching in this graph. Give a reasoning why is that maximum.

   (b) Does this graph contain a vertex cover set whose size is 5?

   (c) Is {1,2,3,4,1,12} a vertex cover set?

   (d) Give a minimum vertex cover set of this graph. Give a reasoning why is that minimum.

7. Let GRAPH DIAMETER be the following decision problem:

   Input: A simple graph G, a non-negative length function $l : E(G) \to R^{+}$ and a number k.

   Question: Is it true that $dist(u, v)$ (the distance between vertices u and v) is at most k for each u, v pair of the vertices?

   Show that GRAPH DIAMETER is in P.

8. Consider the following decision problem:

   Input: An undirected graph G and two of its vertices: u, v.

   Question: Is there a vertex x such that $dist(x, u) \leq 100$ and $dist(x, v) \leq 100$? (dist denotes the graph theoretical distance)

   Show that this decisision problem is in class P.