Combinatorial optimization

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1. Determine whether these functions are in $O(n^2)$.
   (a) $10n^2 + 20n + |\sin(n)|$
   (b) $8n^2 log(n)$
   (c) $1.5n + 3\sqrt{n}$
2. Show that:
   (a) $8n^2 \log_2 n \in O(n^3)$
   (b) $2n - \sqrt{n} \in O(n)$
   (c) $(n+32)(2n^2+12n) \in O(n^3)$
   (d) $(n \log_2(n^2+2) + n^2)(n^3+2) \in O(n^5)$
3. Show that $n! \notin O(n^{100})$.
4. We have two algorithms which solve the same problem. The time complexity of algorithm A is the function $f_A(n)$ and similarly the time complexity of B is $f_B(n)$. We know that $f_A(n) \in O(f_B(n))$. Are these statements true?
   (a) Algorithm A is faster than algorithm B on all possible inputs?
   (b) Algorithm A is faster than algorithm B on all sufficiently large inputs?
5. Consider the following algorithm. A step here is the writing of a *. Show that the time complexity of this algorithm is $O(n^3)$.

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for i=0 to n-1:
        for j=i+1 to n:
                print j pieces of *
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6. Sort these functions to increasing order according to their growing speed. So if f_i preceeds f_j that means that $f_i \in O(f_j)$ and $f_j \notin O(f_i)$.
   $f_1(n) = 8n^3$ $\quad f_2(n) = 5\sqrt{n} + 1000n$ $\quad f_3(n) = 2^{(\log_2 n)^2}$ $\quad f_4(n) = 1514n^2 log_2(n)$
7. Consider the functions $f(n) = 1.5n!$ and $g(n) = 200(n-1)!$. Prove or disprove the following statements:
   (a) $f(n) \in O(g(n))$
   (b) $g(n) \in O(f(n))$
8. Which $a, b > 1$ integers satisfy the following?
   (a) $\log_a n \in O(\log_b n)$
   (b) $2^{an} \in O(2^{bn})$