1. Let $n = p_1 p_2 \cdots p_k$, where every $p_i > 1$ and p_i is a prime, furthermore $p_i \neq p_j$ if $i \neq j$. At most how many divisor of n can be choosen in such a way that no two of them are coprimes?

If k is a divisor, then n/k is also a divisor and they are coprimes in this case. Therefore the maximum number of such divisions is at most half of the divisions, so $2^n/2 = 2^{n-1}$.
2^{n-1} can be attanied if we consider all the divisors which contain p1 as a prime factor.

2. King Arthur sends out a reconing team every day. Each team must consist exactly k knights and two teams cannot be the same. However every two teaam must share a common member. How many days can Arthur sends out a reconing team following these rules?

Lets say that the knights are the numbers from 1 to n. A group is a subset of [n] and the family of groups is k-uniform. Any two group intersect each other, therefore we can use Erdős-Ko-Rado if k<=n/2. In that case the answer is $\binom{n-1}{k-1}$ and a good construction for that is when a knight is contained in all groups.
If k>n/2, then the answer is $\binom{n}{k}$.

3. m lines are given on the plane. Assume that all the planes are not incident to the same point and no two lines are paralell to each other. Prove that these lines determine at least m intersection points.

Since no two lines are paralell to each other, every two lines have an intersection point.
Let the base set be the intersection points, lets denote them with 1,2..n.
Each line can be represented by a set which contains its intersection points.
Any two of these sets intersect each other at one point, so by Fischer's inequality, the

4. A chemical company produces pesticides. They are testing the effect of their new product on m different plants at n different fields. They plant k different plants at each field and they plant each plant at r different fields. For each pair of plants there are exactly $l \geq 1$ fields where both of them have been planted. We know that $l \neq r$. Show that $n \geq m$.

Lets Ai be the set of fields where plant i have been planted $|A_i \cap A_j| = l \geq 1$ if $i \neq j$
Since l≠r, Ai≠Aj if i≠j and {A1,A2,..Am} is a set system. So by fischers inequality m<=n.

5. There are some k element sets and any two of them intersect at l elements. Prove that there is an element which is contained in at most k sets.

Let n denote the number of elements and m denote the number of sets.
Indirectly assume the contrary. That means that every element is included in at least k+1 sets. Each set contains k elements, therefore the total number of sets, m, is at least

$m \geq \frac{(k+1)\cdot n}{k} > n$ but by Fischer's inequality $m \leq n$ ↯, So there must be an element which is included in <=k sets.

6. There is a 100 element set. We choose some of its 20 and 80 element subsets in such a way that any two choosen subset intersect each other. At most how many such subsets can be choosen?

If a 20 element subset is choosen, then its complement is an 80 element subset which is forbidden and vica versa. The total number of these subsets is $\binom{100}{20} + \binom{100}{80} = 2 \cdot \binom{100}{80}$

and we can choose at most half of them, so <=$\binom{100}{80}$.

That bound can be attained if we choose all of the 80 element subsets which of course intersect each other.

7. Show that, if the family $\mathcal{F} \subseteq 2^{[n]}$ contains 2^{n-1} sets and any two of them intersect each other, then there are sets $F_1, F_2 \in \mathcal{F}$ such that $|F_1 \cap F_2| = 1$.

1) $A \in \mathcal{F} \Rightarrow \bar{A} \notin \mathcal{F}$.

IF $\mathcal{F}$ contains a one element set, then its intersection with anything else from $\mathcal{F}$ is itself, therefore in this case we are done. So lets assume that there is no one element set in

Since the size of $\mathcal{F}$ is 2^{n-1} if a set is not included in $\mathcal{F}$ then its complement must be included. Therfore all the n-1 element subsets are included. If there is a 2 element set then its intersection with some n-1 element subsets has size one. So assume that there are no two element subsets. Then all of the n-2 element subsets must be included and if we have a 3 element subset then we are done. So assume that there are no 3 element subsets, then all the n-3 element subsets are included. Etc.