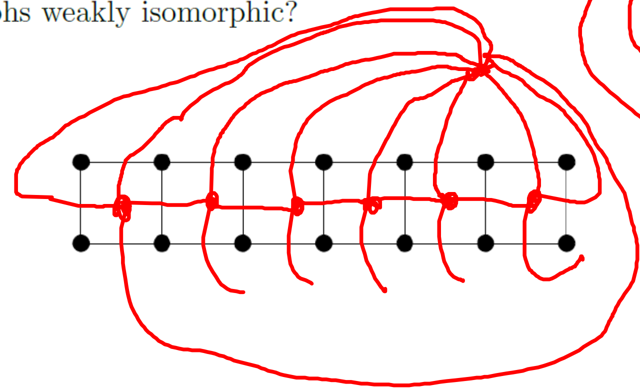
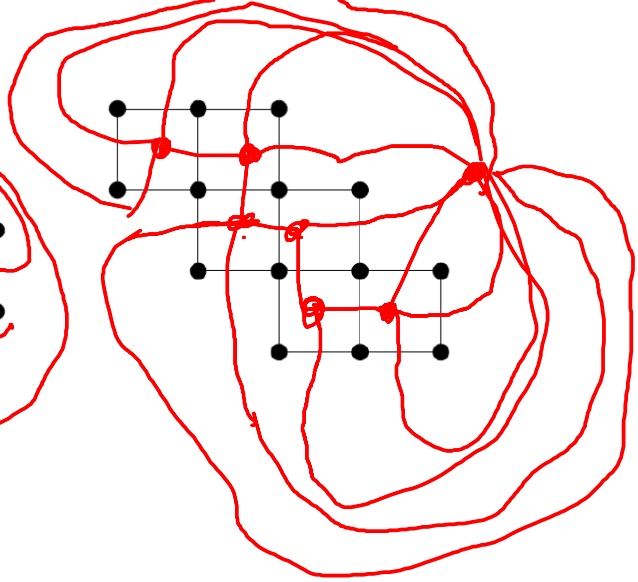
1. Are these graphs weakly isomorphic?





Their duals are isomorphic. So by Witney II the duals of their duals are weakly isomorphic. These original plane graphs are connected, so the dual of their duals are themselves. So we are done and they are weakly isomorphic.

2. Prove that two trees are weakly isomorphic if and only if they have the same amount of vertices.

If they have different amount of vertices, then their edge number is different as well because e=n-1 in a tree. And of course there is no bijection between two sets of different cardinality.

If they have the same amoun of vertices then any bijection between the two edge sets satisfy the definition of week isomorphism because there is no cycle and any edge is a cut-edge.

3. Let $G(V, E)$ be a simple planar graph. Show that E can be partitioned to E_1 and E_2 such that (V, E_1) and (V, E_2) are bipartite graphs.

Pick an optimal coloring of G. Since its chromatic number is at most 4, it uses at most four colors. Let E1 be the set of edges whose endpoints have colors {1,2}, {2,3} or {3,4} and let E2 be the set of edges whose endpoints have colors {1,3},{3,2} or {2,4}. The permutation (23) of the colors maps E1 to E2 so it is enough to show that V,E1 is bipartite. We show that it cannot contain an odd cycle. In any cycle in that graph we do a walk in the path 1,2,3,4 (which are the color classes) and we can get back to our starting position after even number of steps. So we are done.

4. Let graph G and G^* be finite simple graphs. We know that G and G^* are duals of each other. Show that, $\min\{\delta(G), \delta(G^*)\} = 3$, where δ is the minimum degree.

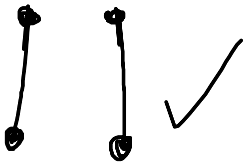
By Withney I, G and G* are planar. Indirectly assume that both minimum degree are 4. It means that G is planar and its minimum degree is 4 and every face of G is bounded by at least 4 edges. Such a planar graph contains at most 2n-4 edges. But in this graph e>=4n/2=2n which is a contradiction.

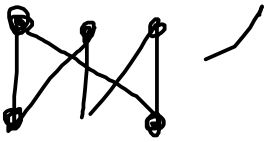
5. Let G be an $n \geq 3$ vertex simple plane graph which has $3n - 6$ edges. What is the maximum degree of the dual of G?

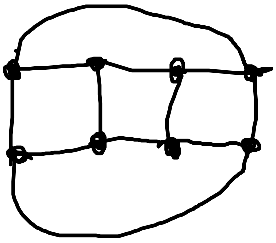
G is a completely triangulated graph, otherwise it has more than 3n-6 edges. So each face is a triangle. Therefore the maximum degree of G* is 3.

6. Let $F_n = K_{n,n} - nK_2$ be the bipartite graph which we can obtain from $K_{n,n}$ by deleting the edges of a perfect matching. For which n is F_n planar?

The number of edges of F(n) is n^2-n. IF F(n) is planar then since its shortest cycle is 4 it must contain at most 4n-4 edges. (the number of vertices is 2n) 4n-4>=n^2-n if n<5. We need to check those cases one by one:

F(2):  planar

F(3)  neither contains a $K_{3,3}$ nor a K_5 or its subdivision so it is pl

F(4)  It is planar.

So for $n<5$ it is planar and for $n>=5$ is not planar.

6. Assume that G is a plane graph, each face of G is a triangle and each face of G^* is a quadrilateral. How many edges and how many vertices does G have?

G is connected, so the dual of its dual is itself. Therefore the degree of each vertex of G is 4 and the degree of each vertex of G^* is 3.
$e=4n/2=2n$ so $n=e/2$ and $e=e^*=3n^*/2=3f/2$, so $f=2/3e$. Puting this into Euler's formula:
$n+f=e+2$ $e/2+2/3e=e+2$ $1/6\ e=2$ so $e=12$. $n=6$.

7. How big can be the chromatic number of a perfect planar graph?

4. K_4 is planar and perfect and its chromatic number is 4. It cannot be bigger by the 4 color theorem.