## Combinatorics and graph theory II. 2022 fall, exam topics

We have proved the framed theorems.

1. Perfect graphs, line graphs, examples for perfect graphs: interval graphs, bipartite graphs, weak perfect graph theorem, strong perfect graph theorem.
2. Partial ordering, poset, chain, antichain, Mirsky's theorem, Dilworth's theorem, comparability graph, connection to perfect graphs, proof of Dilworth's thm by the weak perfect graph theorem
3. Plane graphs, planar graphs, Euler's formula, upper bounds on the edge number of planar graphs, Kuratowski's theorem, proof of the easy direction of Kuratowsi's theorem
4. Topological dual, the properties of the dual and the connections between G and $\mathrm{G}^{*}$, algebric dual, weakly isomorphic graphs, Whitney's theorems (I, $\boxed{I I}$ and III).
5. List coloring, list chromatic number, the connection between $\chi(G)$ and $\operatorname{ch}(G)$, the connection between $\operatorname{ch}(G)$ and $\Delta(G)$, Galvin's theorem, List coloring conjecture, Thomassens's theorem
6. Ramsey numbers, $R(3,3)=6$, Ramsey theorem and its proof by Erdős and Szekeres, upper bound on $R(k, k)$, probabilisctic method, lower bound on $R(k, k)$,
7. $R\left(c_{1}, c_{2}, \ldots c_{t}\right)$, upper bounds on $R\left(c_{1}, c_{2}, \ldots c_{t}\right), R_{k}\left(c_{1}, c_{2}, \ldots c_{t}\right)$, Schur's theorem, Van der Waerden's theorem, Szemerédi's theorem, Erdős-Szekeres theorem (the happy ending problem),
8. k-partite graphs, complete k-partite graphs, $e x(n, H), E x(n, H)$, Turán's theorem, Erdős-Stone theorem, Erdős-Simonovits theorem, $e x(n, H)$ when $H$ is bipartite, Erdős-Kővári-Sós-Turán theorem, proof of the upper bound of Erdős-Kővári-Sós-Turán
9. Set families, hypergraphs, Erdős-Ko-Rado theorem, Fischer's inequality, Ray-Chaudhuri-Wilson theorem,
10. Dual hypergaph, De Bruijn-Erdős theorem, "near pencil" example, Sperner system, Sperner's theorem, LYM inequality
11. Linear recurrence with constant coefficients, Fibonacci numbers, Generating functions, Generating function method, characteristic equation method, Closed-form expression of $F_{n}$, determination of $F_{n}$ by the generating function, determination of $F_{n}$ by the characteristic equation
12. Catalan numbers, several definitions, recurrence for $C_{n}$, closed-form expression of $C_{n}$, determination of $C_{n}$ by the generating function, determination of $C_{n}$ by the mirroring technique
