## Combinatorics and graph theory II. 2022 fall, exam topics

We have proved the framed theorems.

- 1. Perfect graphs, line graphs, examples for perfect graphs: interval graphs, bipartite graphs, weak perfect graph theorem, strong perfect graph theorem.
- 2. Partial ordering, poset, chain, antichain, Mirsky's theorem, Dilworth's theorem, comparability graph, connection to perfect graphs, proof of Dilworth's thm by the weak perfect graph theorem
- 3. Plane graphs, planar graphs, Euler's formula, upper bounds on the edge number of planar graphs Kuratowski's theorem, proof of the easy direction of Kuratowsi's theorem
- 4. Topological dual, the properties of the dual and the connections between G and G<sup>\*</sup>, algebric dual, weakly isomorphic graphs, Whitney's theorems (I, II and III).
- 5. List coloring, list chromatic number, the connection between  $\chi(G)$  and ch(G), the connection between ch(G) and  $\Delta(G)$ , Galvin's theorem, List coloring conjecture, Thomassens's theorem
- 6. Ramsey numbers, R(3,3) = 6, Ramsey theorem and its proof by Erdős and Szekeres, upper bound on R(k,k), probabilisctic method, lower bound on R(k,k),
- 7.  $R(c_1, c_2, \ldots c_t)$ , upper bounds on  $R(c_1, c_2, \ldots c_t)$ ,  $R_k(c_1, c_2, \ldots c_t)$ , Schur's theorem, Van der Waerden's theorem, Szemerédi's theorem, Erdős-Szekeres theorem (the happy ending problem),
- 8. k-partite graphs, complete k-partite graphs, ex(n, H), Ex(n, H), Turán's theorem, Erdős-Stone theorem, Erdős-Simonovits theorem, ex(n, H) when H is bipartite, Erdős-Kővári-Sós-Turán theorem, proof of the upper bound of Erdős-Kővári-Sós-Turán
- 9. Set families, hypergraphs, Erdős-Ko-Rado theorem, Fischer's inequality, Ray-Chaudhuri-Wilson theorem,
- 10. Dual hypergaph, De Bruijn-Erdős theorem, "near pencil" example, Sperner system, Sperner's theorem, LYM inequality
- 11. Linear recurrence with constant coefficients, Fibonacci numbers, Generating functions, Generating function method, characteristic equation method, Closed-form expression of  $F_n$ , determination of  $F_n$  by the generating function, determination of  $F_n$  by the characteristic equation
- 12. Catalan numbers, several definitions, recurrence for  $C_n$ , closed-form expression of  $C_n$ , determination of  $C_n$  by the generating function, determination of  $C_n$  by the mirroring technique