Name: _____

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | $\Sigma$ |
|---|---|---|---|---|---|---|---|---|
| 6/ | 7/ | 6/ | 6/ | 7/ | 6/ | 6/ | 6/ | 50___ |

**Mathematics I. (BSc)– Exam Test 2**
**22th of January, 2014.**

You need reach at least 20 points to pass.

**1.** (6 p.) Find the derivatives of the functions:

a.) $f(x) = \dfrac{x^2 + \cosh^3 4x}{\sqrt{2x-1}}$, b.) $g(x) = \dfrac{1}{\ln(1+e^x)}$, c.) $h(x) = (x^4 + \sin x)^{4x}$.

**2.** (4+3 p.) a.) Show that the lines

$$x = -2 + t, \quad y = 3 + 2t, \quad z = 4 - t$$

and

$$x = 3 - t, \quad y = 4 - 2t, \quad z = t$$

are parallel and find an equation of the plane they determine.

b.) Solve the equation and give the result in algebric form:

$$(\sqrt{3} - i)z^4 - 2i = 0.$$

**3.** (6 p.) Sketch the graph of

$$f(x) = \frac{x^2}{2} + \frac{4}{x}.$$

**4.** (6 p.) Find the following limits, if they exists:

a.) $\lim\limits_{x\to\infty} x \tan^{-1}\dfrac{2}{x}$, b.) $\lim\limits_{x\to\infty} e^{\frac{1}{x}}(\ln(2x) - \ln(x+1))$, c.) $\lim\limits_{x\to 0+}(\tan x)^{\frac{1}{\ln x}}$.

**5.** (7 p.) Let

$$f(x) = \frac{1}{2}(3 - \sin\frac{3\pi - x}{2}).$$

a.) Show that the inverse of $f$ exists and find it.
b.) Find the range of $f(x)$, and the domain and the range of $f^{-1}(x)$.
c.) Sketch the graph of the functions $f(x)$ and $f^{-1}(x)$.
d.) Give the equation of the tangent line of $f(x)$ at $x_0 = 3\pi$.

**6.** (6 p.) Evaluate the indefinit integrals:

a.) $\int \dfrac{1-x}{(3-x)(1+x^2)}dx$, b.) $\int x\ln(1+x^2)dx$.

**7.** (6 p.) Evaluate the definite integrals:

a.) $\int\limits_0^{0.5} 5^{\sqrt{1-2x}}$, b.) $\int\limits_0^{\frac{1}{2}} \arcsin(2x)dx$.

**8.** (6 p.) Find the area between the curves

$$y = e^{-2(x-1)}, \; y = e^{x-1} \text{ and } y = e^2.$$

Name: _____

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | $\Sigma$ |
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**Mathematics I. (BSc)– Exam Test VI.**
**28th of Aug. 2015.**

You need reach at least 20 points to pass.

**1.** (6 p.) Let

$$f(x) = 1 - \sin^{-1}\left(\frac{1-x}{2-x}\right).$$

a.) Find the domain, the range and the derivative of $f(x)$.
b.) Show that the inverse of $f$ exists and find it.
c.) Find the domain, the range and the derivative of the inverse of $f(x)$.

**2.** (3+4 p.) a.) Find an equation for the plane passing through the given point $P(0,1,-1)$ and parallel to the lines

$$e_1 : 2x = y = 3z \quad \text{and} \quad e_2 : 5(x+7) = -2(y-3) = 10(z-4).$$

b.) Solve the equation and give the result in algebric form:

$$(\sqrt{3} - i)z^4 - 2i = 0.$$

**3.** (7 p.) Sketch the graph of

$$f(x) = \frac{x^2 - x + 1}{x - 4}.$$

**4.** (6 p.) Let given

$$f(x) = 3\cos^{-1}\left(x^2 + \frac{\pi}{2}\right) \quad \text{and} \quad g(x) = \tan\frac{1}{x}.$$

a.) $f \circ g(x) = ?$ and $D_{f \circ g} = ?$ b.) $(f \circ g)'(x) = ?$ c.) $\lim_{x \to \infty} f \circ g(x) = ?$

**5.** (6 p.) a.) Determine the 2015th derivatives ($f^{(2015)}(x) = ?$) of the following functions:

$$f(x) = (x^2 - 1)e^{-x}.$$

b.) $\lim_{x \to 1-} f(x) = ?$ c.) $\lim_{x \to -\infty} f(x) = ?$

**6.** (7 p.) Evaluate the integrals:

$$\text{a.)} \int \frac{2x+3}{x^2(x+1)}\,dx, \qquad \text{b.)} \int \sqrt{x^2 - 9}\,dx.$$

**7.** (6 p.) Evaluate the definite integrals:

$$\text{a.)} \int_0^2 \frac{1}{1+e^x}\,dx, \qquad \text{b.)} \int_0^2 \frac{2x-4}{x^2+6x+10}\,dx.$$

**8.** ( 5 p.) Draw the area between the given curves and calculate the value of the area:

$$y = 3 + 2x - x^2, \quad y = 3 - x.$$

Name: _____

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | $\Sigma$ |
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**Mathematics I. (BSc)– Exam Test I.**
**23th of December, 2014.**

You need reach at least 20 points to pass.

**1.** (6 p.) Let

$$f(x) = 1 + \sin^{-1}\left(\frac{1+x}{2+x}\right).$$

a.) Find the domain, the range and the derivative of $f(x)$.
b.) Show that the inverse of $f$ exists and find it.
c.) Find the domain, the range and the derivative of the inverse of $f(x)$.

**2.** (3+4 p.) a.) Find an equation for the plane passing through the given point $P(3,1,-1)$ and parallel to the lines

$$e_1 : 2x = -y = 3z \quad \text{and} \quad e_2 : 5(x-7) = -2(y+3) = 10(z-4).$$

b.) Solve the equation and give the result in algebric form:

$$(\sqrt{3}+i)z^4 + 2i = 0.$$

**3.** (7 p.) Sketch the graph of

$$f(x) = \frac{x^2+x-1}{x+4}.$$

**4.** (6 p.) Let given

$$f(x) = 3\cos\left(x^2 + \frac{\pi}{2}\right) \quad \text{and} \quad g(x) = \tan^{-1}\frac{1}{x}.$$

a.) $f \circ g(x) = ?$ and $D_{f\circ g} = ?$ b.) $(f \circ g)'(x) = ?$ c.) $\lim\limits_{x\to\infty} f \circ g(x) = ?$

**5.** (6 p.) a.) Determine the 2014th derivatives ($f^{(2014)}(x) = ?$) of the following functions:

$$f(x) = (x^2-1)e^x.$$

b.) $\lim\limits_{x\to 1-} f(x) = ?$ c.) $\lim\limits_{x\to-\infty} f(x) = ?$

**6.** (6 p.) Evaluate the integrals:

a.) $\int \frac{2x-3}{x^2(x-1)}\,dx,$ b.) $\int \sqrt{x^2-4}\,dx.$

**7.** (6 p.) Evaluate the definite integrals:

a.) $\int_1^2 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx,$ b.) $\int_{-3}^{-2} \frac{2x+4}{x^2+6x+10}\,dx.$

**8.** ( 6 p.) Draw the area between the given curves and calculate the value of the area supposing $x \geq 0$:

$$y = \frac{2}{1+x^2}, \quad y = x^2.$$

Name: _____

| 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | $\sum$ |
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| 5/ | 7/ | 6/ | 6/ | 8/ | 9/ | 4/ | 5/ | 50___ |

**Mathematics I. (BSc)– Exam Test**
**8th of Okt., 2008.**

**1.** (5 p.) Find an equation of the plane containing the line $x = -2 + 3t$, $y = 4 + 2t$, $z = 3 - t$ and perpendicular to the plane $x - 2y + z = 5$.

**2.** (7 p.) a.) Give the complex numbers $zw$, $\frac{z}{w}$ in algebraic form where $z = 2 + 3i$ and $w = -1 - 2i$ and give the value of $\arg(z + w)$.
c.) Solve the equation $z^6 + 81z^2 = 0$.

**3.** (6 p.) Sketch the graph of $f(x) = \frac{4x}{(x+1)^2}$, and give the equation of the tangent line of $f(x)$ at $x_0 = -2$.

**4.** (6 p.) Find the following limits, if they exists:
a.) $\lim_{x\to 0}(1 + 2x)^{\frac{1}{2\ln x}}$, b.) $\lim_{x\to 0}\frac{1-\cos 4x}{x^2}$, c.) $\lim_{x\to 1+}\left(\frac{1}{x-1} - \frac{1}{\ln x}\right)$.

**5.** (8 p.) Let $f(x) = \frac{\pi}{4} + \cos^{-1}(1 - 4x)$.
a) Find the domain and the range of $f$ and $f^{-1}$.
b) Give the inverse function of it.
c) Find the derivative of the functions $f$ and $f^{-1}$.

**6.** (9 p.) Evaluate the integrals:
a.) $\int \frac{x^3-2}{x^2+2}dx$, b.) $\int \frac{1}{\sqrt{x^2+6x+1}}dx$, c.) $\int x5^{1-2x}dx$.

**7.** (4 p.) Find the limit of each convergent sequence:
a.) $\lim_{n\to\infty}\left(\frac{n-1}{n+3}\right)^{3n}$, b.) $\lim_{n\to\infty}\frac{3+\sin n^2}{n+2}$.

**8.** (5 p.) Find the area between the curves $y = x + 1$, $y = -x^2 + 2x + 3$ and $y = 0$.