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Mathematics II. (BSc)– Exam 2.
Jun 4, 2014.

You need reach at least 20 points to pass.

1. (6 p.) a.) Prove that

$$\underline{a}_1 = (0, 1, 1), \quad \underline{a}_2 = (1, 0, 1), \quad \underline{a}_3 = (1, 1, 0)$$

forms basis in \mathbb{R}^3 .

- b.) Construate an orthonormal basis $\underline{e}_1, \underline{e}_2, \underline{e}_3$, where \underline{e}_1 parallel with \underline{a}_1 .

2. (6 p.) a.) Given the matrix

$$\underline{\underline{A}} = \begin{pmatrix} 3 & 3 \\ 4 & 2 \end{pmatrix}.$$

Find the eigenvalues and eigenvectors of the matrix $\underline{\underline{A}}$.

- b.) What is the eigenvalues and eigenvectors of the matrix $\underline{\underline{B}} = 2\underline{\underline{A}}^2 - 4\underline{\underline{E}}$?

3. (6 p.) Solve the next differential equation using Laplace transform:

$$y'' + 2y' + y = \sin 2x, \quad y(0) = 0, \quad y'(0) = 1.$$

4. (7 p.) Solve the following differential equation:

$$y'' - 2y' + 2y = 25xe^x, \quad y(0) = 1, \quad y'(0) = 0.$$

5. (7 p.) a.) Find the values of the volumen-integral:

$$\iiint_V dV.$$

$V: x^2+y^2 \leq 4, 0 \leq z \leq 16-(x^2+y^2)$

b.) Sketch the region of integration, reverse the order of integration, and evaluate

the integral:

$$\int_{x=0}^1 \int_{y=0}^{3x} e^{6x-y} dx dy + \int_{x=1}^4 \int_{y=0}^3 e^{6x-y} dx dy.$$

6. (6 p.) Let the function

$$f(x, y) = \ln \sqrt{y^2 - x^2} + x^{-y}$$

is an equation of a surface.

- a.) Find the gradient of the function at $P_0(2, 3)$?
- b.) Give the equation of the tangent plane at $P_0(2, 3)$.
- c.) Calculate the directional derivative of $f(x, y)$ at $P_0(2, 3)$
in the direction $\underline{v} = (1, -1)$.

7. (6 p.) Test the function

$$f(x, y) = x^3 - y^3 - 3xy.$$

for local maxima, minima and saddle points.

8. (6 p.) Find the following limit (prove the convergence of uniform):

$$\lim_{x \rightarrow 0} \sum_{n=1}^{\infty} \frac{\cos^2(n^2 x)}{2^{3n+1}}.$$