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Mathematics II. (BSc)– Exam 1.
May 28, 2014.

You need reach at least 20 points to pass.

1. (6 p.) Give the coordinates of the vector $\underline{v} = (0, 2, 3)$ with respect to the basis formed by

the vectors

$$\underline{a}_1 = (4, 0, 1), \quad \underline{a}_2 = (-2, 1, 0), \quad \underline{a}_3 = (-2, 0, 1).$$

2. (6 p.) Let

$$\underline{\underline{A}} = \begin{pmatrix} 4 & -2 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Find the eigenvalues and eigenvectors of the matrix $\underline{\underline{A}}$.

3. (6 p.) Solve the next differential equation using Laplace transform:

$$y'' + 4y = \cos x, \quad y(0) = 0, \quad y'(0) = 1.$$

4. (7 p.) Solve the following differential equation:

$$y'' - 2y' + 2y = 5 \sin x, \quad y(0) = 1, \quad y'(0) = 0.$$

5. (7 p.) a.) Find the values of the double integral:

$$\iint_T 2yx^2 dx dy.$$

$T: 1 \leq x^2 + y^2 \leq 4, 0 \leq x, y, x \leq y \leq \sqrt{3}x$

b.) Sketch the region of integration, reverse the order of integration, and evaluate

the integral:

$$\int_{y=0}^2 \int_{x=0}^y 5xy dx dy + \int_{y=2}^4 \int_{x=0}^{4-y} 5xy dx dy.$$

6. (6 p.) Let the function

$$f(x, y) = \frac{\sin(x - y)}{x^2 - y^2}$$

is an equation of a surface.

- a.) Find the gradient of the function at $P_0(0, \pi)$?
- b.) Give the equation of the tangent plane at $P_0(0, \pi)$.
- c.) Calculate the directional derivative of $f(x, y)$ at $P_0(0, \pi)$ in the direction $\underline{v} = (-8, 6)$.

7. (6 p.) Find the domain of convergence and the sum of the series:

$$\sum_{n=1}^{\infty} \frac{x^{n-1}}{n}.$$

8. (6 p.) Find Taylor series at $x_0 = 0$ for the functions

$$\text{a.) } f(x) = \frac{1}{1 + 3x}, \quad \text{b.) } g(x) = \frac{1}{(1 + 3x)^2}.$$

and give the domain of convergence.