Name: ____

Calculus

Lin.Alg.

1.	2.	3.	4.	$\sum$	5.	6.	7.	8.	$\sum$
4/	7/	6/	8/	25___	8/	6/	5/	6/	25___

Mathematics II. (BSc)– 2nd Midterm Test
9th of May, 2012.

1. **Calculus examples**

   (You need reach at least 8 points to pass this part.)

2. (4 p.) Find the limits if they exist

   a.) $\lim\limits_{(x,y)\to(0,0)} \frac{3x^2+5y^2}{2x^2+y^2}$, b.) $\lim\limits_{(x,y)\to(0,0)} \frac{3xy^3}{2x^2+2y^2}$.

3. (7 p.) Given the function $f(x,y) = e^{y^2-x-1}(2x+1)^5$ and a point $P_o(-1,0)$.

   a.) Find the derivative of f at P_o in the direction of $\underline{v} = -3\underline{i} + 4\underline{j}$.

   b.) Find the direction in which f increases or decreases most rapidly at P_o. Then find the derivatives of f in these directions.

   c.) Find an equation for the tangent plane at the point P_o on the given surface.

4. (6 p.) Given the function $f(x, y) = (2x + y - 4)^2 + (x + y)^2$ and a closed triangular

plate bounded by the lines $x = 0, \quad y = 0, \ x + y = 1$. Find the absolute

maximum and minimum values of f on the given domain.

5. (8 p.) Sketch the region of integration, reverse the order of integration, and evaluate

the integral

$$\int_0^{12} \int_{\sqrt{3x}}^{6} \sqrt{1 + y^3} dy dx.$$

Linear Algebra examples

(You need reach at least 8 points to pass this part.)

5. (8 p.)

$$\text{rank}\begin{pmatrix} t+3 & 5 & 6 \\ -1 & t-3 & -6 \\ 1 & 1 & t+4 \end{pmatrix} = ?$$

6. (6 p.) Give an orthonormal base in V!

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} , \quad x - y + 2z = 0 \right\}$$

7. (5 p.) $\underline{u} = (3, 0, 1, 2)$, $\underline{v} = (-1, 2, 7, -3)$, $\underline{w} = (2, 0, 1, 1)$.

a.) $\|\underline{u} + \underline{v}\| = ?$

b.) $d(\underline{u}, \underline{v}) = ?$

c.) What is the angle between $\underline{u}$ and $\underline{w}$?

6. (6 p.) True or false? (Please, motivate your answer!)

a.) The set of all integer numbers is a linear space.

b.) $V = \left\{ \begin{pmatrix} a\, b \\ 0\, a \end{pmatrix} , \quad a, b \in \mathbb{R} \right\}$ is a subspace in $\mathbb{R}^{3x3}$.

c.) If $\underline{\underline{A}} \in \mathbb{R}^{3x3}$, rank $\left(\underline{\underline{A}}\right)$ = rank $\left(\underline{\underline{A}} \mid \underline{b}\right) = 2$, then the number of the possible solutions in $\underline{\underline{A}}\underline{x} = \underline{b}$ is 1.

d.) $\underline{v}_1 = (1, 2, 3)$, $\underline{v}_2 = (1, 3, -1)$, $\underline{v}_3 = (5, -1, -1)$ form base in $\mathbb{R}^3$.

e.) $\langle \underline{u}, \underline{v} \rangle = 2u_1v_1 + 3u_2v_2 + 4u_3v_3$ is a scalar product in $\mathbb{R}^3$.

f.) $\underline{u} = (k, k, 1)$, $\underline{v} = (k, 5, 6)$ are orthogonal together if and only if $k = -2$.