Linear Algebra Methods for Forbidden Configurations

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Abstract

We say a matrix is *simple* if it is a (0,1)-matrix with no repeated columns. Given m and a $k \times l$ (0,1)-matrix F we define $\mathrm{forb}(m, F)$ as the maximum number of columns in a simple m-rowed matrix A for which no $k \times l$ submatrix of A is a row and column permutation of F. For all k-rowed F (simple or non-simple) Füredi has shown that $\mathrm{forb}(m, F)$ is $O(m^k)$. We are able to determine for which k-rowed F we have that $\mathrm{forb}(m, F)$ is $O(m^{k-1})$ and for which k-rowed F we have that $\mathrm{forb}(m, F)$ is $\Theta(m^k)$.

We need a bound for a particular choice of F. Define D_{12} to be the $k \times (2^k - 2^{k-2} - 1)$ (0,1)-matrix consisting of all nonzero columns on k rows that do not have $\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]$ in rows 1 and 2. Let $\mathbf{0}$ denote the column of k 0's. Define $F_k(t)$ to be the concatenation of $\mathbf{0}$ with $t + 1$ copies of D_{12}. We are able to show that $\mathrm{forb}(m, F_k(t))$ is $\Theta(m^{k-1})$. Linear algebra methods and indicator polynomials originated in this context in a paper of the authors and Füredi and Sali. We provide a novel application of these methods.

The results are further evidence for the conjecture of Anstee and Sali on the asymptotics for fixed F of $\mathrm{forb}(m, F)$.

Keywords: VC-dimension, forbidden configurations

1 Introduction

It is convenient to use the language of matrix theory and of sets. We define a *simple* matrix as a (0,1)-matrix with no repeated columns. Let $[m] = \{1, 2, \ldots, m\}$. For a

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