Word-representable graphs

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A graph $G = (V, E)$ is said to be (word-)representable if there exists a word W over the alphabet V such that u and v are adjacent if and only if the occurrences of u and v in W alternate. These graphs were originally introduced in relation to the free spectrum of the Perkis semigroup.

We obtain a number of structural results on representable graphs. We give an alternative representation of these graphs as those that can be oriented to satisfy a certain semi-transitivity property. This allows us to characterize the types of graphs that are representable, in particular showing that they include 3-colorable graphs.

The graph is k-representable if it is representable by a word where each letter occurs k times. The representation number of a graph G is the smallest k such that G is k-representable. We show that the representation number is at most $n = |V|$, while there are graphs with representation number as large as $n/2$. A corollary is that deciding if a graph is representable is in NP; on the other hand, it is still open if it is in P.

This is joint work with Artem Pyatkin (Sobolev Institute of Mathematics) and Sergey Kitaev (Reykjavik University).