SPT\((q, k, n)\)-codes

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The following was motivated by database research. We consider \(q\)-ary codes of length \(n\) with property

- \(C\) has minimum Hamming-distance at least \(n - k + 1\).
- For any set of \(k - 1\) coordinates there exist two codewords that agree exactly there.

A \(k - 1\)-set of coordinate can be considered as a 'direction', so in \(C\) the minimum distance is \textit{attained in all directions}. Such a code \(C\) is called \textit{sphere-packing type code} of parameters \((q, k, n)\), or \(SPT(q, k, n)\)-code for short. For example, the rows of the \(k+1 \times k+1\) identity matrix form an \(SPT(2, k, k+1)\)-code.

\textbf{Definition 1} Let \(q > 1\) and \(k > 1\) be given natural numbers. Let \(f(q, k)\) be the maximum \(n\) such that there exists an \(SPT(q, k, n)\)-code.

New bounds on \(f(q, k)\) are proven. Upper bound employs spherical codes, the lower bound is probabilistic construction using Lovász Local Lemma.