Consistency of Natural Relations on Sets

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The natural relations for sets are those definable in terms of the emptiness of the subsets corresponding to Boolean combinations of the sets. For pairs of sets, there are just five natural relations of interest, namely, strict inclusion in each direction, disjointness, intersection with the universe being covered, or not.

Let N denote $\{1, 2, ..., n\}$ and $\binom{n}{2}$ denote $\{(i, j) \mid i, j \in N \text{ and } i < j\}$. A function g on $\binom{n}{2}$ specifies one of these relations for each pair of indices. Then g is said to be consistent on $M \subseteq N$ if and only if there exists a collection of sets corresponding to indices in M such that the relations specified by g hold between each associated pair of the sets.

It is proved that if g is consistent on all subsets of N of size three then g is consistent on N. It is also shown that the result concerning binary natural relations can be generalized to r-ary natural relations for arbitrary $r \geq 2$.