Control in Computational Social Choice

Jiehua Chen[1] , **Joanna Kaczmarek**[2] , **Paul Nüsken**[2] , **Jörg Rothe**[2] , **Ildikó Schlotter**[3] and **Tessa Seeger**[2]
[1]TU Vienna, Vienna, Austria
[2]Institut für Informatik, MNF, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, Germany
[3]Institute of Economics, HUN-REN Centre for Economic and Regional Studies, 1097 Budapest, Hungary
jiehua.chen@ac.tuwien.ac.at, {joanna.kaczmarek, paul.nuesken, rothe, tessa.seeger}@hhu.de, schlotter.ildiko@krtk.hun-ren.hu

Abstract

We survey the notion of control in various areas of computational social choice: in voting, judgment aggregation, fair division, cooperative game theory, matching under preferences, group identification, and opinion diffusion. In all these scenarios, control can be exerted, e.g., by adding or deleting agents with the goal of influencing the outcome.

1 Introduction

Computational social choice was founded by three seminal papers of Bartholdi *et al.* [1989b; 1989a; 1992], and the founding fathers of this area—at that time new, but now a key topic at all large AI conferences—focused on winner determination, manipulation, and control of elections. We survey some central models and results about control in computational social choice since its beginnings. Control in elections means that a (usually external) agent (called the election chair) modifies the structure of an election by, e.g., adding or deleting voters or candidates with the goal of either making a favorite candidate win (in the constructive case) or preventing a despised candidate's victory (in the destructive case).

Along with manipulation and bribery [Faliszewski *et al.*, 2009a], control attacks on single-winner elections were the main focus of attention in the early days of computational social choice and the subject of book chapters [Baumeister and Rothe, 2024; Faliszewski and Rothe, 2016; Conitzer and Walsh, 2016]. Since then the study of control has spread like a wildfire over various other subfields of computational social choice. Our survey covers control not only in single-winner and multiwinner voting but also in judgment aggregation, fair division, cooperative game theory, matching under preferences, group identification, and opinion diffusion. In each of these fields, we describe the underlying models and scenarios and explain how control can be exerted in them, for instance, by adding or deleting agents with the goal of influencing the outcome. We give an overview of some of the main results on control in each of these fields and highlight a number of challenges for future research.

2 Control in Voting

An election is given as a pair (C, V) with a set C of candidates and a list V of votes over C. We will assume that votes are linear orders (but note that there are also other ways of representing voter preferences, e.g., approval ballots). In order to determine the winner(s) of an election (respectively, its winning committee(s) of a given size), many single-winner (respectively, multiwinner) voting rules have been proposed, see, e.g., [Baumeister and Rothe, 2024; Zwicker, 2016; Brams and Fishburn, 2002] (respectively, [Baumeister *et al.*, 2024; Faliszewski *et al.*, 2017]). We start with the former.

2.1 Single-Winner Voting Rules

A very important class of single-winner voting rules are the *positional scoring protocols* where candidates score points based on their positions in the votes. Among these, we focus on *plurality* where only top-ranked candidates score a point, and on the rule by *Borda* [1781] where each candidate ranked in the i-th position of a vote scores $m - i$ points for m candidates. For instance, in the election shown in Figure 1, d with a score of 5 is the plurality winner (whereas a, b, and c score only 1, 3, and 3 points) and b and d with a score of 19 are the Borda winners (whereas a and c score only 17 points).

Other voting rules are based on pairwise comparisons of candidates—among those, the *Condorcet-consistent* rules are particularly important: rules that elect the Condorcet winner whenever there exists one. A *Condorcet winner* of an election is a candidate who beats all other candidates by a majority of votes in pairwise comparison. Condorcet winners do not always exist [Condorcet, 1785], but if so, they are unique. For example, the voting rule due to *Schulze* [2011] is Condorcet-consistent. Being widely used in practice and celebrated for its many useful properties, it is based on the strength of paths between candidates in the *weighted majority graph (WMG)* of an election (C, V): There is a vertex for each candidate, and there is an edge from x to y exactly if the edge weight, defined as the difference $D_V(x, y)$ of how many voters prefer x to y minus how many prefer y to x, is positive (see the WMGs in Example 1). Define the *path strength* $\mathrm{str}(p)$ as the weight $D_V(c, d)$ of the weakest edge (c, d) on p. For each pair of distinct candidates $c, d \in C$, define the *strength of a strongest path between c and d* as $P_V(c, d) = \max\{\mathrm{str}(p) \mid p$ is a path from c to $d\}$. Now, $c \in C$ is a *Schulze winner of* (C, V) if $P(c, d) \geq P(d, c)$ for each $d \in C \setminus \{c\}$.

We now describe some typical control scenarios in voting.

Example 1. Anna (a), Belle (b), Chris (c), and David (d) run for president of the renowned Association for Advancing

$v_1 : b\ a\ c\ d$	$v_2 : d\ c\ b\ a$
$v_3 : c\ a\ b\ d$	$v_4 : d\ b\ c\ a$
$v_5 : a\ d\ b\ c$	$v_6 : c\ b\ a\ d$
$v_7 : b\ c\ a\ d$	$v_8 : d\ a\ b\ c$
$v_9 : d\ b\ a\ c$	$v_{10} : c\ a\ d\ b$
$v_{11} : d\ c\ a\ b$	$v_{12} : b\ a\ d\ c$

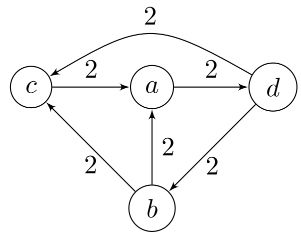
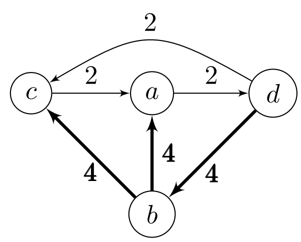


Figure 1: An election (left) and its WMG (right)

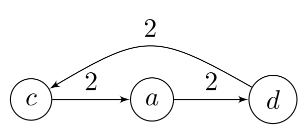
Anonymous Ideas (AAAI). The 12 current AAAI members eligible to vote cast the ballots shown in Figure 1 (left), where candidates are ordered from left (most preferred) to right (least preferred). The corresponding WMG (right) shows that there is no Condorcet winner (as no vertex has only outgoing edges) and all candidates are Schulze winners. Evil Eve, though, is not happy about this. Being the election chair, she has the power to add new voters (whose preferences she knows). Wishing to make her favorite candidate d the *unique* Schulze winner, she adds the (boldfaced) voters $v_{13}, \ldots, v_{18}$, and we obtain the following new election and WMG:

$v_1 : b\ a\ c\ d$	$v_2 : d\ c\ b\ a$
$v_3 : c\ a\ b\ d$	$v_4 : d\ b\ c\ a$
$v_5 : a\ d\ b\ c$	$v_6 : c\ b\ a\ d$
$v_7 : b\ c\ a\ d$	$v_8 : d\ a\ b\ c$
$v_9 : d\ b\ a\ c$	$v_{10} : c\ a\ d\ b$
$v_{11} : d\ c\ a\ b$	$v_{12} : b\ a\ d\ c$
$\boldsymbol{v_{13}} : b\ a\ c\ d$	$\boldsymbol{v_{14}} : d\ c\ b\ a$
$\boldsymbol{v_{15}} : b\ c\ a\ d$	$\boldsymbol{v_{16}} : d\ a\ b\ c$
$\boldsymbol{v_{17}} : d\ b\ a\ c$	$\boldsymbol{v_{18}} : c\ a\ d\ b$



The WMG above shows that Eve has reached her goal: d alone wins. Fraudulent Frodo, however, is not amused. By making Eve's control attack public (thus causing her impeachment), he becomes the new election chair. Unlike Eve's constructive goal, his goal is purely destructive: He doesn't care who wins as long as d is *not* the only Schulze winner. Of course, he cannot simply delete the voters just added, but he can exert control by deleting candidates (except d). By deleting b, he obtains the following new election and WMG:

$v_1 : a\ c\ d$	$v_2 : d\ c\ a$
$v_3 : c\ a\ d$	$v_4 : d\ c\ a$
$v_5 : a\ d\ c$	$v_6 : c\ a\ d$
$v_7 : c\ a\ d$	$v_8 : d\ a\ c$
$v_9 : d\ a\ c$	$v_{10} : c\ a\ d$
$v_{11} : d\ c\ a$	$v_{12} : a\ d\ c$
$v_{13} : a\ c\ d$	$v_{14} : d\ c\ a$
$v_{15} : c\ a\ d$	$v_{16} : d\ a\ c$
$v_{17} : d\ a\ c$	$v_{18} : c\ a\ d$



Now, Frodo has reached his goal: Each of a, c, and d win, so d is not a unique Schulze winner. ⌟

The scenarios described in Example 1 give rise to defining the following problems. For *constructive control by adding voters* (CCAV), we are given a set C of candidates, two lists (V and U) of votes over C, where already registered voters cast the votes in V and in U are those of as yet unregistered voters, a designated candidate $c \in C$, and a positive integer k. The question is whether there is a sublist $U' \subseteq U$, $|U'| \leq k$, such that c wins the election $(C, V \cup U')$. For *destructive control by deleting candidates* (DCDC), an election (C, V), a designated candidate $c \in C$, and a positive integer k are given, and we ask whether at most k candidates can be deleted from C such that c does not win the resulting election.

Two winner models distinguished: The *unique-winner model* requires c to be the only winner in the constructive case and not winning alone in the destructive case, whereas the *nonunique-winner model* only requires c to be one (of possibly several) winner(s) in the constructive case and not winning at all in the destructive case. The problems of *constructive control by deleting voters* (CCDV) and *by adding or deleting candidates* (CCAC[1] and CCDC) and of *destructive control by adding candidates* (DCAC) and *by adding or deleting voters* (DCAV and DCDV) are defined analogously.

A variety of *control scenarios by partition of voters or candidates* have also been studied; especially control by partition of voters is interesting, as it models "gerrymandering"—ways of redistricting voting districts. Due to space limitations, however, we omit them here and only mention that some cases of destructive control by partition of candidates collapse [Hemaspaandra *et al.*, 2020; Carleton *et al.*, 2023]. We also omit defining and discussing further types of control, such as control by *replacing voters or candidates* [Loreggia *et al.*, 2015; Erdélyi *et al.*, 2021] and certain *"more natural models" of control by partition* [Erdélyi *et al.*, 2015c] as well as *multimode control attacks* [Faliszewski *et al.*, 2011], which combine various standard control types. All these control attacks omitted here are discussed in detail by Baumeister and Rothe [2024] and Faliszewski and Rothe [2016].

For some of the control scenarios defined above, the election chair's goal can never be reached. For example, constructive control by adding candidates is never possible for the chair in Condorcet voting: If the designated candidate c is not a Condorcet winner in a given election, c does not beat all other candidates in pairwise comparison, so c can never be made a Condorcet winner by adding more candidates. We then say Condorcet voting is *immune* (I) to this type of control. If a voting rule is not immune to some control type, we say it is *susceptible* (S) to it, and in that case we consider the computational complexity of the corresponding problem. If it can be solved in P, we say the rule is *vulnerable* (V) to this control type; and if it is NP-hard, we say the rule is *resistant* (R) to it. Table 1 gives an overview of the known complexity results for the four rules and the eight control scenarios defined above. Results marked by $*$ are due to Bartholdi *et al.* [1992]; by † due to Hemaspaandra *et al.* [2007]; by § due to Russel [2007]; by \$ due to Elkind *et al.* [2011]; by ♠ due to Parkes and Xia [2012]; by ‡ due to

[1]Bartholdi *et al.* [1992] originally defined a variant (CCAUC) where an *unlimited* number of candidates may be added: No k is given. For most voting rules, CCAUC behaves just as CCAC in terms of complexity. Interestingly, however, they behave differently for Llull voting [Hägele and Pukelsheim, 2001]: CCAUC can be solved in P, yet CCAC is NP-complete [Faliszewski *et al.*, 2009b].

Menton and Singh [2013]; by ❄ due to Chen *et al.* [2015]; by £ due to Loreggia *et al.* [2015]; by ♡ due to Hemaspaandra and Schnoor [2016]; by ♣ due to Neveling and Rothe [2021]; by ◇ originally claimed by Menton and Singh [2013] whose proof was later corrected by Maushagen *et al.* [2024]; and by ¶ originally claimed by Menton and Singh [2012] but later stated as open [Menton and Singh, 2013] and re-established by Maushagen *et al.* [2024].

Table 1: Control complexity results for some voting rules

	CCAV	DCAV	CCDV	DCDV	CCAC	DCAC	CCAUC	DCAUC	CCDC	DCDC
Plurality	V*	V†	V*	V†	R†	R†	R*	R†	R*	R†
Borda	R§	V§	R$^{\heartsuit}$	V§	R$^{\$}$	V$^{\pounds}$	R$^{\clubsuit}$	V$^{\clubsuit}$	R$^{\text{❄}}$	V$^{\pounds}$
Condorcet	R*	V†	R*	V†	I†	V†	I*	V†	V*	I†
Schulze	R$^{\spadesuit}$	R$^{\spadesuit}$	R$^{\spadesuit}$	R$^{\spadesuit}$	R$^{\spadesuit}$	?	R‡	?	R$^{\diamondsuit}$	V¶

Observe that the destructive case never is harder than the constructive case (note that immunity means that the control problem is trivial and thus in P). Solving the open cases in Table 1 for Schulze voting seems to be a challenging task.

Challenge 1. *Solve the open cases in Table 1 for Schulze voting: What is the complexity of* DCAC *and* DCAUC*?*

The control complexity has also been explored for many other natural voting rules, which we will not define here: for approval voting [Hemaspaandra *et al.*, 2007]; Copeland and Llull voting [Faliszewski *et al.*, 2009b]; sincere-strategy preference-based approval voting [Erdélyi *et al.*, 2009]; k-approval and k-veto [Lin, 2011; Lin, 2012]; range voting and normalized range voting [Menton, 2013]; ranked-pairs voting [Hemaspaandra *et al.*, 2013; Maushagen *et al.*, 2024]; Bucklin and fallback voting [Erdélyi *et al.*, 2015a; Erdélyi *et al.*, 2015b]; and veto and maximin [Maushagen and Rothe, 2016; Maushagen and Rothe, 2018; Maushagen and Rothe, 2020].

2.2 Multiwinner Voting Rules

3 Control in Judgment Aggregation

4 Control in Fair Allocation

Dividing resources among a set of agents in a fair and efficient way is a practical problem that has been around since biblical times. The possible settings are widely varied, based on the type of resources, the fairness and efficiency criteria, and the possible additional constraints on the desired allocation. The resources to be allocated are usually non-homogeneous, i.e., different agents may value a given part of it differently, and can be either *divisible* or *indivisible*. In *cake cutting*, each agent has a utility function over a divisible resource called the *cake*, while in *fair division*, each agent has utilities over a set of indivisible *items*, expressed either as a cardinal utility function or a linear preference order.

Example 2. Suppose that the Anna, Belle, Chris, and David receive a gift bag from their aunt for Christmas containing a kite (K), a toy lion (L), a pair of mittens (M), a jar of nutella (N), and an oboe (O). To distribute the gifts in a fair way, their father asks the children to evaluate them, eliciting the following values.

	K	L	M	N	O
a:	6	2	1	10	1
b:	0	3	3	10	4
c:	5	3	0	10	2
d:	3	2	0	10	5

	π_{dad}	π_{mum}
a:	K	K
b:	M	L, M
c:	L	P
d:	O	O

The children's father, anticipating a calamity, quickly confiscates the jar of nutella. He allocates the gifts according to the allocation π_{dad} shown above. Pointing out that each child has received a gift that is worth more than the fourth of the total value of all remaining gifts (i.e., more than $\frac{10}{4}$), he walks away with the nutella.

Immediately, a skirmish breaks out, because Belle envies the oboe from David, and Anna envies the kite from Chris. The children's mother comes to the rescue brandishing a set of paints (P), valued to 5 by each child, and redistributes the gifts according to allocation π_{mum} above. Peace ensues. ⌟

In Example 2, the control action performed by the father was *item removal*, with the aim of achieving an allocation that is *proportional*, that is, allocates to each agent p a subset of the item set I having value at least $u_p(I)/|A|$ where A is the set of agents and $u_p : I \to \mathbb{N}$ denotes the valuation function of p which naturally extends to 2^I by assuming additive valuations. The second control action, performed by the mother, was *item addition*, to facilitate an *envy-free* allocation, i.e., allocation $\pi : A \to 2^I$ where $u_p(\pi(p)) \geq u_p(\pi(q))$ holds for each two agents p and q.

The study of control in fair allocation was initiated by Aziz *et al.* [2016]. Besides item removal and item addition, Aziz *et al.* also defined *agent removal* and *agent addition*, as well as *item/agent replacement* and *item/agent partitioning* for achieving fairness. Instead of defining these control actions formally, we will focus on the control action considered most often (in fact, almost exclusively) in the literature: item removal. The popularity of this notion is probably due to the fact that donating goods is a natural and practically feasible option in most scenarios.

Caragiannis *et al.* [2019] considered cardinal and additive preferences, and proposed an algorithm for finding an allocation that is *envy-free up to any item* (EFX), meaning that no agent envies another agents' bundle after the worst item is discarded from it, and is guaranteed to have at least half of the Nash welfare[2] achievable by any allocation. Chaudhury *et al.* [2021] gave a method for finding an allocation that is EFX by donating a bundle of at most $|A|$ items such that no agent would prefer the donated bundle to its own.

In a setting with ordinal preferences, Brams *et al.*[2014] have devised an algorithm for two agents that produces an envy-free[3] partial allocation with the minimal number of unallocated (or from a different perspective, donated) items.

[2]The Nash welfare of an allocation is the geometric mean of the utility values obtained by the agents.

[3]An allocation π is *envy-free* under ordinal preferences, if for each two agents p and q there is an injection f from $\pi(j)$ to $\pi(i)$ such that for each item $x \in \pi(j)$ agent i prefers $f(x)$ to x.

Aziz *et al.* [2016] showed that a similar algorithm is not possible for three agents, since even determining whether a *complete* envy-free allocation exists is NP-hard. Under ordinal preferences, deciding the existence of a *proportional*[4] allocation is easy, but computing the minimum number of items whose removal leaves an instance admitting a proportional allocation is NP-hard already for three agents [Dorn *et al.*, 2021]. Besides obtaining an FPT algorithm for three agents, parameterized by the number of item removals, Dorn *et al.* [2021] also considered a setting where some fixed allocation is given in advance, and the task is to make this allocation proportional by removing items.

Applying control to make an a priori fixed allocation fair was also studied by Boehmer *et al.* [2024] for the setting with additive cardinal utilities over indivisible items, and by Segal-Halevi [2022] for cake cutting with geometric constraints.

Finally, let us mention that researchers have also considered hiding information [Hosseini *et al.*, 2020; Bliznets *et al.*, 2024] to achieve fairness which may also be considered a form of control.

5 Control in Cooperative Game Theory

One of the type of games in cooperative game theory are *coalitional games with a characteristic function* [von Neumann and Morgenstern, 1944], which are defined as a pair (N, v) with *player set* N and *characteristic function* $v : 2^N \to \mathbb{R}$. Each subset of N is called a *coalition*. The coalitional game is *monotonic* if for any C, C' with $C \subseteq C' \subseteq N$, $v(C) \leq v(C')$. It is called *simple*, if additionally $v(C) \in \{0, 1\}$ for all $C \subseteq N$; if $v(C) = 1$, we call C a *winning coalition*, while if $v(C) = 0$, we call it a *losing coalition*. We call a player i *pivotal for* $C \subseteq N \setminus \{i\}$ if $v(C \cup \{i\}) - v(C) = 1$.

The analysis of simple games includes, i.a., answering the question of how import a player is in forming winning coalitions. This importance is measured by *power indices* such as the *Shapley–Shubik index* [Shapley and Shubik, 1954] and the *probabilistic Penrose–Banzhaf index* [Dubey and Shapley, 1979]; both are #P-complete [Valiant, 1979] to compute [Deng and Papadimitriou, 1994; Prasad and Kelly, 1990]. The power indices count—each in a different way—the coalitions for which the player is pivotal in the considered game.

Next to measuring the power, the interesting topic of tempering it has also been wildly studied—we discuss some examples of control in two classes of simple games in the following two subsections.

5.1 Adding or Deleting Players in Weighted Voting Games

A *weighted voting game* (WVG) $\mathcal{G} = (w_1, \ldots, w_n; q)$ is a simple coalitional game with player set N, which consists of a *quota* $q \in \mathbb{N}$ and nonnegative integer weights, where w_i is the *weight of player* $i \in N$. Let $w_C = \sum_{i \in C} w_i$ for $C \subseteq N$. The characteristic function v of $\mathcal{G}$ is defined as follows: $v(C) = 1$ if $w_C \geq q$, and $v(C) = 0$ otherwise.

[4] An allocation $\pi : A \to 2^I$ is *proportional* under ordinal preferences, if for any $i \leq |I|$ each agent gets at least $i/|A|$ items among the first i items of her preference list.

Inspired by the idea discussed in Section 2, Rey and Rothe [2018] introduced control of the player set in a given weighted voting game in two forms: by adding players and by deleting players. The goal of the control, in both cases, is increasing, nondecreasing, decreasing, nonincreasing, or maintaining a given player's power. They analyzed the problems in the context of their computational complexity, so they defined the related decision problems as follows: In the case of control by deleting players, for a given WVG, a distinguished player, and a specified limit, the question is whether it is possible to change or maintain—according to the chosen goal—this player's power index by deleting not more than the specified number of players; in the case of the control by adding players, a set of new players is additionally provided and the question is whether the same goal can be achieved by adding not more than the specified number of new players.

These problems have been studied for the probabilistic Penrose–Banzhaf and the Shapley–Shubik indices [Rey and Rothe, 2018; Kaczmarek and Rothe, 2024b; Kaczmarek and Rothe, 2024c; Kaczmarek and Rothe, 2024a]. For all problems of control by adding players, completeness for the class $\mathrm{NP}^{\mathrm{PP}}$ (the class of problems solvable by a nondeterministic Turing machine accessing PP oracle [Turing, 1939; Gill, 1977]) was established [Rey and Rothe, 2018; Kaczmarek and Rothe, 2024c; Kaczmarek and Rothe, 2024a]. For the problems of control by deleting players, the question of where their completeness can be found is still open; the current known bounds are—depending on the specific problem—are as follows: $\mathrm{NP}^{\mathrm{PP}}$ as the upper bound for all of them [Rey and Rothe, 2018]; the classes NP, coNP, DP [Papadimitriou and Yannakakis, 1984], and Θ_2^p [Papadimitriou and Zachos, 1983] as the lower bounds [Rey and Rothe, 2018; Kaczmarek and Rothe, 2024b].

Kaczmarek and Rothe [2024b] introduced the model of weighted voting games where a game's quota is not directly given. Specifically, it is defined as $r \cdot w_N$ for player set N and a specified factor $r \in [0, 1]$. At first glance, this change does not seem to be significant; however, it is in the context of the problems discussed above. The authors introduced and analyzed the corresponding decision problems, where the control types applied to a given WVG can result in a WVG with not only a modified player set but also an altered quota. They established the upper bound for all of them for $\mathrm{NP}^{\mathrm{PP}}$ and proved their hardness for the classes NP, coNP, and DP.

Challenge 2. *Find a completeness result for the decision problems of control by adding or deleting players with the goal of either changing or maintaining a player's power in both models of WVGs, where it has not been established yet. Analyze these control types for other power indices.*

5.2 Adding or Deleting Edges in Graph-Restricted Weighted Voting Games

WVGs have also been studied with an additional restriction by a graph [Myerson, 1977; Napel *et al.*, 2012]—undirected simple graph (see, e.g., [Diestel, 2017]), whose vertices correspond to players. In these games, coalition C wins if and only if there exists $C' \subseteq C$ such that $w_{C'} \geq q$, for a given quota q, and C' induces a connected subgraph of the restricting graph. Despite the restriction, however, computing the two power indices stays #P-complete [Skibski *et al.*, 2015].

With the new structural component, additional control possibilities arise: for a given graph-restricted WVG, a distinguished player, and a specified limit, the question is whether adding or removing up to the specified number of edges can change or maintain the player's power [Kaczmarek and Rothe, 2021]. While no completeness results have been established, all these problems are at least NP-hard or coNP-hard. Therefore, the challenges in the analysis of their complexity are analogous to Challenge 2.

6 Control in Matching under Preferences

Most work on control in relation to matching under preferences concentrates on the classic STABLE MARRIAGE (SM) problem and its generalization, the COLLEGE ADMISSION (CA) problem [Gale and Shapley, 1962]. In an instance of SM, we are given a set of agents on a two-sided market, traditionally called *men* and *women*, and a preference list for each agent, which is a strict linear order over a subset of agents from the opposite side of the market. The task is to find a *matching* between men and women that is *stable*, i.e., contains no man–woman pair such that both of them prefer each other to their partner in the matching (called a *blocking pair*).

Example 3. Suppose that Anna (a), Belle (b), Chris (c), and David (d) are attending a dance class, and need to form opposite-sex couples. Their preferences are as follows:

$$\begin{array}{ll} a : c\ d, & c : a\ b, \\ b : c\ d, & d : b\ a. \end{array}$$

The only stable matching in this instance is $\{(a,c),(b,d)\}$. However, Belle's friend, Evil Eve, is among the teachers of the class, and her goal is to match Belle with her top-choice partner, Chris. She considers three options to achieve her goal: (1) declaring that Chris is too short for Anna and hence cannot dance with her, (2) stepping on Anna's toes with her high heels, thereby sending her off to ER, or (3) inviting her attractive friend, Frodo (f) whom Anna prefers to Chris. She decides on option (3), and obtains the following instance:

$$\begin{array}{ll} a : f\ c\ d, & c : a\ b, \\ b : c\ d\ f, & d : b\ a, \\ & f : b\ a. \end{array}$$

But Eve is not entirely satisfied, as now there are two stable matchings, $M_1 = \{(a,f),(b,c)\}$ and $M_2 = \{(a,c),(b,f)\}$. Hence, she declares that Frodo is too tall for Belle, thus ensuring that M_1 becomes the only stable matching. ⌟

Example 3 highlights some of the settings examined by Boehmer *et al.* [2021] who initiated the study of control problems in relation to stable matchings. Boehmer *et al.* defined five manipulative actions and three different goals, thereby obtaining 15 different computational problems. Among these actions are the control actions showcased in Example 3:

- AddAg: adding agents (e.g., inviting Frodo),
- DelAg: deleting agents (e.g., removing Anna from the class), and
- DelAcc: deleting acceptability (e.g., declaring constraints on who can dance with whom);

others involve changing the preference lists of the agents and thus fall into the category of manipulation or bribery.

Example 3 depicts also some of the possible goals that an external controller may want to ensure. These may either focus on a distinguished agent or pair of agents, or aim to obtain a stable matching with some desirable property, e.g., a *perfect* matching, where every agent is matched. In the generalization of SM where the underlying graph is not necessarily bipartite, called the STABLE ROOMMATES model, a stable matching may not exist, so ensuring the existence of a stable matching becomes a meaningful aim. Below, we summarize known results for problems where the controller's aim is that

- MA: a given agent is matched in a stable matching,
- MP: a given pair is contained in a stable matching,
- SM: a given matching becomes stable,
- USM: a given matching becomes the unique stable matching,
- ∃SM: a stable matching exists, or
- ∃PSM: a perfect and stable matching exists.

For each control action $\mathcal{A} \in \{\mathsf{AddAg}, \mathsf{DelAg}, \mathsf{DelAcc}\}$ and each goal $\mathcal{G} \in \{\mathsf{MA}, \mathsf{MP}, \mathsf{SM}, \mathsf{USM}, \exists\mathsf{SM}, \exists\mathsf{PSM}\}$ discussed above, the CONTROL IN STABLE MARRIAGE-$\mathcal{A}$-$\mathcal{G}$ (or CSM-$\mathcal{A}$-$\mathcal{G}$) problem asks for the minimum number of control actions necessary to achieve the given goal in a given SM instance; we call the non-bipartite variants of these problems CONTROL IN STABLE ROOMMATES-$\mathcal{A}$-$\mathcal{G}$ (or CSR-$\mathcal{A}$-$\mathcal{G}$).[5]

Table 2: Complexity results for CONTROL IN STABLE MARRIAGE (or ROOMMATES)-$\mathcal{A}$-$\mathcal{G}$ problems. The problems CSM-$\mathcal{A}$-∃SM are omitted, as a stable matching always exists in an instance of SM. Background coloring is used to group each problem SCR-$\mathcal{A}$-$\mathcal{G}$ and its bipartite variant.

	CSM-$\mathcal{A}$-MA	CSR-$\mathcal{A}$-MA	CSM-$\mathcal{A}$-MP	CSR-$\mathcal{A}$-MP	CSM-$\mathcal{A}$-SM	CSR-$\mathcal{A}$-SM	CSM-$\mathcal{A}$-USM	CSR-$\mathcal{A}$-USM	CSR-$\mathcal{A}$-∃SM	CSM-$\mathcal{A}$-∃PSM	CSR-$\mathcal{A}$-∃PSM
AddAg	R$^{\triangledown}$	R$^{\triangledown}$	R*	R*	V*	R$^{\triangledown}$	R*	R*	R$^{\triangledown}$	R$^{\triangledown}$	R$^{\triangledown}$
DelAg	V$^{\underline{*}}$	V$^{\triangledown}$	V*	V$^{\triangledown}$	R*	R*	R*	R*	V†	V†	V†
DelAcc	R$^{\heartsuit}$	R$^{\heartsuit}$	R*	R*	V*	V$^{\underline{*}}$	V*	?	R$^{\diamond}$	R$^{\spadesuit}$	R$^{\spadesuit}$

In Table 2, result by Boehmer *et al.* [2021] and easy consequences thereof are marked with $*$ and with $\underline{*}$, respectively. Results by Chen and Schlotter [2025] are marked with $\triangledown$, by Tan [1990; 1991] with †, by Schlotter and Mnich [2020] with $\heartsuit$, by Abraham *et al.* [2006] with $\diamond$, and by Biró *et al.* [2010] with ♠. We remark that some of the problems CSM-DelAg-$\mathcal{G}$ have been studied in a the context of *almost stable* matchings, i.e., matchings with few blocking pairs:

[5]We remark that the definition of CSR-$\mathcal{A}$-$\mathcal{G}$ for control goals SM and USM is somewhat problematic for control actions AddAg and DelAg, since these actions change the set of agents. We adopt the approach by Boehmer *et al.* [2021] who assume that the matching M given in the input covers the total set of agents (including all addable agents), and we aim to add a few agents such that the resulting instance contains a matching $M' \subseteq M$ as a stable (or the unique stable) matching.

e.g., an instance of STABLE ROOMMATES admits a matching with at most k blocking pairs if and only if we can ensure the existence of a stable matching by k deletions of acceptability. Similarly, the minimum number of blocking pairs in any matching that matches a given agent or contains a given pair is exactly the minimum number of DelAcc actions that ensures the corresponding goal $\mathcal{G} \in \{\mathsf{MA}, \mathsf{MP}\}$. Further intractability and parameterized results were provided by Gupta and Jain [2025] for weighted and destructive variants of many of the problems in Table 2, and by Bérczi *et al.* [2024] regarding agent deletion problems with additional constraints. Kamiyama [2025] looked at the problem where preferences can contain ties, and we aim for a super-stable matching by deleting as few agents as possible.

A prominent line of research has also emerged in connection to the CA problem, the many-to-one variant of SM where the two sides of the market represent *students* and *colleges*, and each college comes with a *capacity*; the task is then to find a matching of students to colleges that respects capacities and is *stable*, i.e., there exists no student–college pair (s, c) such that s prefers to be matched to c, while either c is unsaturated, or there is a student matched to c to whom c prefers s. The control actions focused on by most researchers in this setting are *capacity increase, capacity decrease* or *capacity modification* (when both increasing and decreasing capacities is allowed). The controller's goal is most often to ensure the existence of a stable matching that fulfills some desirable property such as being perfect or Pareto-optimal, by using as few control actions as possible.

Bobbio *et al.* [2022] have studied the problem of minimizing capacity increase (or decrease) for obtaining a stable matching that minimizes the average college rank to which students are matched, and proved both problems to be NP-complete and hard to approximate; a further study developed mixed integer programs for these problems [Bobbio *et al.*, 2023]. Chen and Csáji [2023] initiated the study of determining the existence of a stable matching that is simultaneously perfect or Pareto-optimal through capacity increases, while bounding the sum or the maximum of these modifications. Among these four problems, only one turned out to be polynomial-time solvable (when we bound the maximum capacity increases and aim for a perfect and stable matching), while the other three are NP-hard. They further investigated the parameterized complexity. Afacan *et al.* [2024] also considered obtaining a Pareto-optimal and stable matching, but they worked with a model that includes a lower quota for each college and has an upper bound only on the sum of college capacities. Gokhale *et al.* [2024] gave a polynomial-time algorithm for stabilizing a given matching (SM) via capacity increase or decrease, and proved that the problem of ensuring that some student–college pair is contained in a stable matching (MP) is NP-hard for these two control actions.Nguyen and Vohra [2018] considered a setting where students can form couples and submit joint preference lists, and showed that there is a capacity modification yielding a stable matching where each hospital's capacity is modified by at most two, and the total capacity modification is at most four.

7 Control in Group Identification

Broadly speaking, group identification deals with finding a *socially qualified group* among individuals. To this end, each individual either qualifies or disqualifies all other individuals (and themselves) for inclusion in the group. More formally, given a set A of individuals and a profile $\varphi : A \times A \to \{0, 1\}$, we use a group identification rule F to determine a socially qualified group $F(\varphi, A) \subseteq A$. We say individual $a \in A$ socially qualifies $b \in A$ if $\varphi(a, b) = 1$ and socially disqualifies b if $\varphi(a, b) = 0$.

Recently, in the setting of control, three group identification rules have been studied. The *consent rule* $f^{(s,t)}$ [Samet and Schmeidler, 2003] and two procedural rules [Dimitrov *et al.*, 2007], namely the *consensus-start-respecting-rule* f^{CSR} and the *liberal-start-respecting-rule* f^{LSR}. In the consent rule $f^{(s,t)}$ a social qualification is determined by their own individual assessment and two thresholds s and t. If an individual qualifies themselves, they are qualified if and only if at least $s - 1$ other individuals qualify them. Vice versa, if an individual does not qualify themselves, they are disqualified if and only if at least $t - 1$ other individuals also disqualify them.

The second type of group identification rules are the procedural rules. These rules recursively add individuals who are qualified by the current members into the group until no member qualifies any individual outside the group (i.e., no new member was added during a recursion call). The rules differ in the selection of the initial group. The liberal-start-respecting-rule f^{LSR} starts with the set of individuals, who qualify themselves, while in the consensus-start-respecting-rule f^{CSR}, the initial set is given by individuals who are qualified by everyone (including themselves).

Example 4. After noticing the rigged election, the Association for Advancing Anonymous Ideas (AAAI) proposes a new format for finding their leadership. They ask each of the four candidates (Anna, Belle, Chris and David) who they deem qualified to lead the association in the coming years. Of course, everyone qualifies themselves. In addition, Anna qualifies everyone; Belle qualifies David; Chris qualifies Belle and David; and, lastly, David qualifies Chris. The resulting qualifications are depicted in Figure 2. Using f^{CSR} as the group identification rule, the association determines Belle and David as the qualified group. Evil Eve, again unhappy with the result and wielding the power to change the process, removes Chris from contention. Now, David is the sole qualified individual. This, again, results in Frodo taking action and adding Grace to the pool of candidates. Grace qualifies only herself and is deemed qualified by everyone (the new resulting qualification graph is depicted in Figure 3). As a result, the new socially qualified group consists solely of Grace.

⌟

Evil Eve and Frodo both changed the qualified group by influencing the structure of the identification task. The study of control complexity in group identification was initiated by Yang and Dimitrov [2018], who first studied *constructive group control by adding individuals* (CGCAI), *constructive*

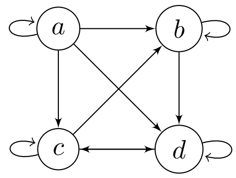


Figure 2: Social qualifications before control is exerted in Example 4. The social qualification graph contains a directed edge from v to v' if and only if $\varphi(v, v') = 1$ for $v, v' \in A$.

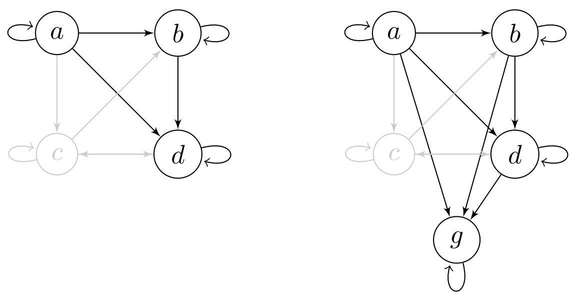


Figure 3: Social qualifications after control is excerted in Example 4. The left graph shows the qualifications after Evil Eves control action and the graph on the right depicts the state after Frodo's intervention.

group control by deleting individuals (CGCDI) and *constructive group control by partitioning of individuals* (CGCPI). In the setting of group identification, the election chair's goal is to help a subset $A^+ \subseteq A$ of individuals to become socially qualified (constructive control) [Yang and Dimitrov, 2018] or prevent a subset $A^- \subseteq A$ of individuals from being included in the socially qualified group (destructive control) [Erdélyi *et al.*, 2020].

Table 3 gives an overview of results for control in group identification for f^{LSR} and f^{CSR}. Results marked by $\diamond$ are due to Yang and Dimitrov [2018], and by $\spadesuit$ are due to Erdélyi et al. [2020].

Table 3: Control complexity results for group identification for f^{LSR} and f^{CSR}.

	CGCAI	DGCAI	CGCDI	DGCDI	CGCPI	DGCPI
f^{CSR}	R$^\diamond$	R$^\spadesuit$	V$^\diamond$	V$^\spadesuit$	?	?
f^{LSR}	R$^\diamond$	I$^\spadesuit$	I$^\diamond$	V$^\spadesuit$	I$^\diamond$	V$^\spadesuit$

Recently, a domain restriction, namely the domain of *consecutive qualifications*, has been given more attention. Yang and Dimitrov [2023; 2024] studied control with the restriction to the consecutive domain for the consent rule, both procedural rules and through the lens of parameterized complexity. In addition to control, Erdélyi et al. [2020] also studied *bribery* in group identification. Boehmer et al. [2023] initiated the study of a generalized problem (*Constructive+Destructive*) and *Exact Control* in bribery, as well as considering these in the setting of parameterized complexity. A different bribery setting, namely *mircrobribery*, was studied by Erdélyi and Yang [2020].

Challenge 3. *Solve the open cases in Table 3 for f^{CSR}: What is the complexity of* CGCPI *and* DGCPI*?* [6]

8 Control in Opinion Diffusion

9 Conclusions and Outlook

Ethical Statement

There are no ethical issues.

Acknowledgments

This work was supported in part by DFG research grant RO-1202/21-2 (project number 438204498), by the Vienna Science and Technology Fund (WWTF) [10.47379/VRG18012]. Schlotter is supported by the Hungarian Academy of Sciences under its Momentum Programme (LP2021-2) and its János Bolyai Research Scholarship.

References

[Abraham *et al.*, 2006] D.J. Abraham, P. Biró, and D.F. Manlove. Almost stable matchings in the Roommates problem. In *Proceedings of the 3rd Workshop on Approximation and Online Algorithms*, pages 1–14. Springer, *Lecture Notes in Computer Science #3879*, 2006.

[Afacan *et al.*, 2024] Mustafa Oğuz Afacan, Umut Dur, and Martin Van der Linden. Capacity design in school choice. *Games and Economic Behavior*, 146:277–291, 2024.

[Aziz *et al.*, 2016] H. Aziz, I. Schlotter, and T. Walsh. Control of fair division. In *Proceedings of the 25th International Joint Conference on Artificial Intelligence*, pages 67–73. AAAI Press/IJCAI, July 2016.

[Bartholdi III *et al.*, 1989a] J. Bartholdi III, C. Tovey, and M. Trick. The computational difficulty of manipulating an election. *Social Choice and Welfare*, 6(3):227–241, 1989.

[Bartholdi III *et al.*, 1989b] J. Bartholdi III, C. Tovey, and M. Trick. Voting schemes for which it can be difficult to tell who won the election. *Social Choice and Welfare*, 6(2):157–165, 1989.

[Bartholdi III *et al.*, 1992] J. Bartholdi III, C. Tovey, and M. Trick. How hard is it to control an election? *Mathematical and Computer Modelling*, 16(8/9):27–40, 1992.

[Baumeister and Rothe, 2024] D. Baumeister and J. Rothe. Preference aggregation by voting. In J. Rothe, editor, *Economics and Computation. An Introduction to Algorithmic Game Theory, Computational Social Choice, and Fair Division*, Classroom Companion: Economics, chapter 4, pages 233–367. Springer, 2nd edition, 2024.

[6] Erdélyi et al. [2020] show that f^{CSR}-DGCPI is vulnerable under the restriction that a qualifies b iff b qualifies a for all $a, b \in A$.

[Baumeister *et al.*, 2024] D. Baumeister, P. Faliszewski, J. Rothe, and P. Skowron. Multiwinner voting. In J. Rothe, editor, *Economics and Computation. An Introduction to Algorithmic Game Theory, Computational Social Choice, and Fair Division*, Classroom Companion: Economics, chapter 6, pages 403–465. Springer, 2nd edition, 2024.

[Bérczi *et al.*, 2024] Kristóf Bérczi, Gergely Csáji, and Tamás Király. Manipulating the outcome of stable marriage and roommates problems. *Games and Economic Behavior*, 147:407–428, 2024.

[Biró *et al.*, 2010] P. Biró, D.F. Manlove, and S. Mittal. Size versus stability in the marriage problem. *Theoretical Computer Science*, 411(16–18):1828–1841, 2010.

[Bliznets *et al.*, 2024] Ivan Bliznets, Anton Bukov, and Danil Sagunov. Fair division with minimal withheld information in social networks. *Theoretical Computer Science*, 991:114446, 2024.

[Bobbio *et al.*, 2022] Federico Bobbio, Margarida Carvalho, Andrea Lodi, and Alfredo Torrico. Capacity variation in the many-to-one stable matching. Technical Report arXiv: 2205.01302 [cs.CC], ACM Computing Research Repository (CoRR), 2022.

[Bobbio *et al.*, 2023] Federico Bobbio, Margarida Carvalho, Andrea Lodi, Ignacio Rios, and Alfredo Torrico. Capacity planning in stable matching: An application to school choice. In *Proceedings of the 24th ACM Conference on Economics and Computation*, page 295, 2023.

[Boehmer *et al.*, 2021] Niclas Boehmer, Robert Bredereck, Klaus Heeger, and Rolf Niedermeier. Bribery and control in stable marriage. *Journal of Artificial Intelligence Research*, 71:993–1048, 2021.

[Boehmer *et al.*, 2023] N. Boehmer, R. Bredereck, D. Knop, and J. Luo. Fine-grained view on bribery for group identification. *Autonomous Agents and Multi-Agent Systems*, 37(1):21, Mar 2023.

[Boehmer *et al.*, 2024] Niclas Boehmer, Robert Bredereck, Klaus Heeger, Dušan Knop, and Junjie Luo. Multivariate algorithmics for eliminating envy by donating goods. *Autonomous Agents and Multi-Agent Systems*, 38:43, 2024.

[Borda, 1781] J.-C. de Borda. Mémoire sur les élections au scrutin. *Histoire de L'Académie Royale des Sciences, Paris*, 1781. English translation appears in the paper by de Grazia [**?**].

[Brams and Fishburn, 2002] S. Brams and P. Fishburn. Voting procedures. In K. Arrow, A. Sen, and K. Suzumura, editors, *Handbook of Social Choice and Welfare*, volume 1, chapter 4, pages 173–236. North-Holland, 2002.

[Brams *et al.*, 2014] S. Brams, M. Kilgour, and C. Klamler. Two-person fair division of indivisible items: An efficient, envy-free algorithm. *Notices of the AMS*, 61(2):130–141, 2014.

[Caragiannis *et al.*, 2019] I. Caragiannis, N. Gravin, and X. Huang. Envy-freeness up to any item with high Nash welfare: The virtue of donating items. In *Proceedings of the 20th ACM Conference on Economics and Computation*, pages 527–545. ACM Press, June 2019.

[Carleton *et al.*, 2023] B. Carleton, M. Chavrimootoo, L. Hemaspaandra, D. Narváez, C. Taliancich, and H. Welles. Separating and collapsing electoral control types. In *Proceedings of the 22nd International Conference on Autonomous Agents and Multiagent Systems*, pages 1743–1751. IFAAMAS, May/June 2023.

[Chaudhury *et al.*, 2021] B. Chaudhury, T. Kavitha, K. Mehlhorn, and A. Sgouritsa. A little charity guarantees almost envy-freeness. *SIAM Journal on Computing*, 50(4):1336–1358, 2021.

[Chen and Csáji, 2023] Jiehua Chen and Gergely Csáji. Optimal capacity modification for many-to-one matching problems. In *Proceedings of the 22nd International Conference on Autonomous Agents and Multiagent Systems*, pages 2880–2882. IFAAMAS, 2023.

[Chen and Schlotter, 2025] J. Chen and I. Schlotter. Controlling Stable Marriage and Stable Roommates: Complexity and algorithms. Technical Report ????, ACM Computing Research Repository (CoRR), 2025. Available on-line at http://arxiv.org/abs/????

[Chen *et al.*, 2015] J. Chen, P. Faliszewski, R. Niedermeier, and N. Talmon. Elections with few voters: Candidate control can be easy. In *Proceedings of the 29th AAAI Conference on Artificial Intelligence*, pages 2045–2051. AAAI Press, January 2015.

[Condorcet, 1785] J.-A.-N. de Caritat, Marquis de Condorcet. Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix, 1785. Facsimile reprint of original published in Paris, 1972, by the Imprimerie Royale. English translation appears in I. McLean and A. Urken, *Classics of Social Choice*, University of Michigan Press, 1995, pages 91–112.

[Conitzer and Walsh, 2016] V. Conitzer and T. Walsh. Barriers to manipulation in voting. In F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A. Procaccia, editors, *Handbook of Computational Social Choice*, chapter 6, pages 127–145. Cambridge University Press, 2016.

[Deng and Papadimitriou, 1994] X. Deng and C. Papadimitriou. On the complexity of cooperative solution concepts. *Mathematics of Operations Research*, 19(2):257–266, 1994.

[Diestel, 2017] R. Diestel. *Graph Theory*. Springer Berlin Heidelberg, 2017.

[Dimitrov *et al.*, 2007] D. Dimitrov, S. Sung, and Y. Xu. Procedural group identification. *Mathematical Social Sciences*, 54(2):137–146, 2007.

[Dorn *et al.*, 2021] B. Dorn, R. de Haan, and I. Schlotter. Obtaining a proportional allocation by deleting items. *Algorithmica*, 83(5):1559–1603, 2021.

[Dubey and Shapley, 1979] P. Dubey and L. Shapley. Mathematical properties of the Banzhaf power index. *Mathematics of Operations Research*, 4(2):99–131, 1979.

[Elkind *et al.*, 2011] E. Elkind, P. Faliszewski, and A. Slinko. Cloning in elections: Finding the possible winners. *Journal of Artificial Intelligence Research*, 42:529–573, 2011.

[Erdelyi and Yang, 2020] G. Erdelyi and Y. Yang. Microbribery in group identification. In *Proceedings of the 19th International Conference on Autonomous Agents and MultiAgent Systems*, AAMAS '20, page 1840–1842, Richland, SC, 2020. International Foundation for Autonomous Agents and Multiagent Systems.

[Erdélyi *et al.*, 2009] G. Erdélyi, M. Nowak, and J. Rothe. Sincere-strategy preference-based approval voting fully resists constructive control and broadly resists destructive control. *Mathematical Logic Quarterly*, 55(4):425–443, 2009.

[Erdélyi *et al.*, 2015a] G. Erdélyi, M. Fellows, J. Rothe, and L. Schend. Control complexity in Bucklin and fallback voting: A theoretical analysis. *Journal of Computer and System Sciences*, 81(4):632–660, 2015.

[Erdélyi *et al.*, 2015b] G. Erdélyi, M. Fellows, J. Rothe, and L. Schend. Control complexity in Bucklin and fallback voting: An experimental analysis. *Journal of Computer and System Sciences*, 81(4):661–670, 2015.

[Erdélyi *et al.*, 2015c] G. Erdélyi, E. Hemaspaandra, and L. Hemaspaandra. More natural models of electoral control by partition. In *Proceedings of the 4th International Conference on Algorithmic Decision Theory*, pages 396–413. Springer-Verlag *Lecture Notes in Artificial Intelligence #9346*, September 2015.

[Erdélyi *et al.*, 2020] G. Erdélyi, C. Reger, and Y. Yang. The complexity of bribery and control in group identification. *Autonomous Agents and Multi-Agent Systems*, 34(1):8, 2020.

[Erdélyi *et al.*, 2021] G. Erdélyi, M. Neveling, C. Reger, J. Rothe, Y. Yang, and R. Zorn. Towards completing the puzzle: Complexity of control by replacing, adding, and deleting candidates or voters. *Journal of Autonomous Agents and Multi-Agent Systems*, 35(2):41:1–41:48, 2021.

[Faliszewski and Rothe, 2016] P. Faliszewski and J. Rothe. Control and bribery in voting. In F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A. Procaccia, editors, *Handbook of Computational Social Choice*, chapter 7, pages 146–168. Cambridge University Press, 2016.

[Faliszewski *et al.*, 2009a] P. Faliszewski, E. Hemaspaandra, and L. Hemaspaandra. How hard is bribery in elections? *Journal of Artificial Intelligence Research*, 35:485–532, 2009.

[Faliszewski *et al.*, 2009b] P. Faliszewski, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe. Llull and Copeland voting computationally resist bribery and constructive control. *Journal of Artificial Intelligence Research*, 35:275–341, 2009.

[Faliszewski *et al.*, 2011] P. Faliszewski, E. Hemaspaandra, and L. Hemaspaandra. Multimode control attacks on elections. *Journal of Artificial Intelligence Research*, 40:305–351, 2011.

[Faliszewski *et al.*, 2017] P. Faliszewski, P. Skowron, A. Slinko, and N. Talmon. Multiwinner voting: A new challenge for social choice theory. In U. Endriss, editor, *Trends in Computational Social Choice*, chapter 2, pages 27–47. AI Access Foundation, 2017.

[Gale and Shapley, 1962] D. Gale and L. Shapley. College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15, 1962.

[Gill, 1977] J. Gill. Computational complexity of probabilistic Turing machines. *SIAM Journal on Computing*, 6(4):675–695, 1977.

[Gokhale *et al.*, 2024] Salil Gokhale, Samarth Singla, Shivika Narang, and Rohit Vaish. Capacity modification in the stable matching problem. In *Proceedings of the 23rd International Conference on Autonomous Agents and Multiagent Systems*, pages 697–705. IFAAMAS, May 2024.

[Gupta and Jain, 2025] Sushmita Gupta and Pallavi Jain. Manipulation with(out) money in matching market. In *adt24*, page 273–287. Springer, *Lecture Notes in Computer Science #15248*, 2025.

[Hägele and Pukelsheim, 2001] G. Hägele and F. Pukelsheim. The electoral writings of Ramon Llull. *Studia Lulliana*, 41(97):3–38, 2001.

[Hemaspaandra and Schnoor, 2016] E. Hemaspaandra and H. Schnoor. Dichotomy for pure scoring rules under manipulative electoral actions. In *Proceedings of the 22nd European Conference on Artificial Intelligence*, pages 1071–1079. IOS Press, August/September 2016.

[Hemaspaandra *et al.*, 2007] E. Hemaspaandra, L. Hemaspaandra, and J. Rothe. Anyone but him: The complexity of precluding an alternative. *Artificial Intelligence*, 171(5–6):255–285, 2007.

[Hemaspaandra *et al.*, 2013] L. Hemaspaandra, R. Lavaee, and C. Menton. Schulze and ranked-pairs voting are fixed-parameter tractable to bribe, manipulate, and control. In *Proceedings of the 12th International Conference on Autonomous Agents and Multiagent Systems*, pages 1345–1346. IFAAMAS, May 2013.

[Hemaspaandra *et al.*, 2020] E. Hemaspaandra, L. Hemaspaandra, and C. Menton. Search versus decision for election manipulation problems. *ACM Transactions on Computation Theory*, 12(1):3:1–3:42, 2020.

[Hosseini *et al.*, 2020] Hadi Hosseini, Sujoy Sikdar, Rohit Vaish, Hejun Wang, and Lirong Xia. Fair division through information withholding. In *aaai20*, volume 34, pages 2014–2021. AAAI Press, 2020.

[Kaczmarek and Rothe, 2021] J. Kaczmarek and J. Rothe. Manipulation in communication structures of graph-restricted weighted voting games. In *Proceedings of the 7th International Conference on Algorithmic Decision Theory*, pages 194–208. Springer-Verlag *Lecture Notes in Artificial Intelligence #13023*, November 2021.

[Kaczmarek and Rothe, 2024a] J. Kaczmarek and J. Rothe. Control by adding players to change or maintain the Shapley–Shubik or the Penrose–Banzhaf power index in weighted voting games is complete for $\mathrm{NP}^{\mathrm{PP}}$. In *Proceedings of the 27th European Conference on Artificial Intelligence*, volume 392 of *Frontiers in Artificial Intelligence and Applications*, pages 3525–3532. IOS Press, October 2024.

[Kaczmarek and Rothe, 2024b] J. Kaczmarek and J. Rothe. Controlling weighted voting games by deleting or adding players with or without changing the quota. *Annals of Mathematics and Artificial Intelligence*, 92(3):631–669, 2024.

[Kaczmarek and Rothe, 2024c] J. Kaczmarek and J. Rothe. $\mathrm{NP}^{\mathrm{PP}}$-completeness of control by adding players to change the Penrose–Banzhaf power index in weighted voting games (extended abstract). In *Proceedings of the 23rd International Conference on Autonomous Agents and Multiagent Systems*, pages 2318–2320. IFAAMAS, May 2024.

[Kamiyama, 2025] Naoyuki Kamiyama. Modifying an instance of the super-stable matching problem. *Information Processing Letters*, 189:106549, 2025.

[Lin, 2011] A. Lin. The complexity of manipulating k-approval elections. In *Proceedings of the 3rd International Conference on Agents and Artificial Intelligence*, pages 212–218. SciTePress, January 2011.

[Lin, 2012] A. Lin. *Solving Hard Problems in Election Systems*. PhD thesis, Rochester Institute of Technology, Rochester, NY, USA, March 2012.

[Loreggia *et al.*, 2015] A. Loreggia, N. Narodytska, F. Rossi, B. Venable, and T. Walsh. Controlling elections by replacing candidates or votes (extended abstract). In *Proceedings of the 14th International Conference on Autonomous Agents and Multiagent Systems*, pages 1737–1738. IFAAMAS, May 2015.

[Maushagen and Rothe, 2016] C. Maushagen and J. Rothe. Complexity of control by partitioning veto and maximin elections and of control by adding candidates to plurality elections. In *Proceedings of the 22nd European Conference on Artificial Intelligence*, pages 277–285. IOS Press, August/September 2016.

[Maushagen and Rothe, 2018] C. Maushagen and J. Rothe. Complexity of control by partitioning veto elections and of control by adding candidates to plurality elections. *Annals of Mathematics and Artificial Intelligence*, 82(4):219–244, 2018.

[Maushagen and Rothe, 2020] C. Maushagen and J. Rothe. The last voting rule is home: Complexity of control by partition of candidates or voters in maximin elections. In *Proceedings of the 24th European Conference on Artificial Intelligence*, volume 325 of *Frontiers in Artificial Intelligence and Applications*, pages 163–170. IOS Press, August/September 2020.

[Maushagen *et al.*, 2024] C. Maushagen, D. Niclaus, P. Nüsken, J. Rothe, and T. Seeger. Toward completing the picture of control in Schulze and ranked pairs elections. In *Proceedings of the 33rd International Joint Conference on Artificial Intelligence*, pages 2940–2948. ijcai.org, August 2024.

[Menton and Singh, 2012] Curtis Menton and Preetjot Singh. Manipulation and control complexity of schulze voting. Technical Report arXiv:1206.2111v1 [cs.GT], ACM Computing Research Repository (CoRR), June 2012.

[Menton and Singh, 2013] C. Menton and P. Singh. Control complexity of Schulze voting. In *Proceedings of the 23rd International Joint Conference on Artificial Intelligence*, pages 286–292. AAAI Press/IJCAI, August 2013.

[Menton, 2013] C. Menton. Normalized range voting broadly resists control. *Theory of Computing Systems*, 53(4):507–531, 2013.

[Myerson, 1977] R. Myerson. Graphs and cooperation in games. *Mathematics of Operations Research*, 2(3):225–229, 1977.

[Napel *et al.*, 2012] S. Napel, A. Nohn, and J. Alonso-Meijide. Monotonicity of power in weighted voting games with restricted communication. *Mathematical Social Sciences*, 64(3):247–257, 2012.

[Neveling and Rothe, 2021] M. Neveling and J. Rothe. Control complexity in Borda elections: Solving all open cases of offline control and some cases of online control. *Artificial Intelligence*, 298:103508, 2021.

[Nguyen and Vohra, 2018] Thành Nguyen and Rakesh Vohra. Near-feasible stable matchings with couples. *American Economic Review*, 108(11):3154–3169, 2018.

[Papadimitriou and Yannakakis, 1984] C. Papadimitriou and M. Yannakakis. The complexity of facets (and some facets of complexity). *Journal of Computer and System Sciences*, 28(2):244–259, 1984.

[Papadimitriou and Zachos, 1983] C. Papadimitriou and S. Zachos. Two remarks on the power of counting. In *Proceedings of the 6th GI Conference on Theoretical Computer Science*, pages 269–276. Springer-Verlag *Lecture Notes in Computer Science #145*, 1983.

[Parkes and Xia, 2012] D. Parkes and L. Xia. A complexity-of-strategic-behavior comparison between Schulze's rule and ranked pairs. In *Proceedings of the 26th AAAI Conference on Artificial Intelligence*, pages 1429–1435. AAAI Press, July 2012.

[Prasad and Kelly, 1990] K. Prasad and J. Kelly. NP-completeness of some problems concerning voting games. *International Journal of Game Theory*, 19(1):1–9, 1990.

[Rey and Rothe, 2018] A. Rey and J. Rothe. Structural control in weighted voting games. *The B.E. Journal on Theoretical Economics*, 18(2):1–15, 2018.

[Russel, 2007] N. Russel. Complexity of control of Borda count elections. Master's thesis, Rochester Institute of Technology, 2007.

[Samet and Schmeidler, 2003] D. Samet and D. Schmeidler. Between liberalism and democracy. *Journal of Economic Theory*, 110(2):213–233, 2003.

[Schlotter and Mnich, 2020] Ildikó Schlotter and Matthian Mnich. Stable marriage with covering constraints:

A complete computational trichotomy. *Algorithmica*, 82(1):1136–1188, 2020.

[Schulze, 2011] M. Schulze. A new monotonic, clone-independent, reversal symmetric, and Condorcet-consistent single-winner election method. *Social Choice and Welfare*, 36(2):267–303, 2011.

[Segal-Halevi, 2022] Erel Segal-Halevi. Redividing the cake. *Autonomous Agents and Multi-Agent Systems*, 36:14, 2022.

[Shapley and Shubik, 1954] L. Shapley and M. Shubik. A method of evaluating the distribution of power in a committee system. *The American Political Science Review*, 48(3):787–792, 1954.

[Skibski *et al.*, 2015] O. Skibski, T. Michalak, Y. Sakurai, and M. Yokoo. A pseudo-polynomial algorithm for computing power indices in graph-restricted weighted voting games. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence*, pages 631–637. AAAI Press/IJCAI, July 2015.

[Tan, 1990] J.J.M. Tan. A maximum stable matching for the roommates problem. *BIT Numerical Mathematics*, 30:631–640, 1990.

[Tan, 1991] J.J.M. Tan. Stable matchings and stable partitions. *International Journal of Computer Mathematics*, 39:11–20, 1991.

[Turing, 1939] A. Turing. Systems of logic based on ordinals. *Proceedings of the London Mathematical Society*, s2-45(1):161–228, 1939.

[Valiant, 1979] L. Valiant. The complexity of computing the permanent. *Theoretical Computer Science*, 8(2):189–201, 1979.

[von Neumann and Morgenstern, 1944] J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, 1944.

[Yang and Dimitrov, 2018] Y. Yang and D. Dimitrov. How hard is it to control a group? *Autonomous Agents and Multi-Agent Systems*, 32(5):672–692, Sep 2018.

[Yang and Dimitrov, 2023] Y. Yang and D. Dimitrov. Group control for consent rules with consecutive qualifications. *Mathematical Social Sciences*, 121:1–7, 2023.

[Yang and Dimitrov, 2024] Y. Yang and D. Dimitrov. Group control for procedural rules: parameterized complexity and consecutive domains. *Frontiers of Computer Science*, 18(3):183402, Jan 2024.

[Zwicker, 2016] W. Zwicker. Introduction to the theory of voting. In F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A. Procaccia, editors, *Handbook of Computational Social Choice*, chapter 2, pages 23–56. Cambridge University Press, 2016.