Some open problems in parameterized complexity

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Dagstuhl Seminar 17041
Schloss Dagstuhl, Germany
January 23, 2017

Even Set

Input: Set system $\mathcal{S}$ over a universe U, integer k.
Find: A *nonempty* set $X \subseteq U$ of size at most k such that $|X \cap S|$ is even for every $S \in \mathcal{S}$.

Essentially equivalent formulations:

- With graphs and neighborhoods.
- Minimum circuit in a binary matroid.
- Mininum distance in a linear code over a binary alphabet.

FPT approximation

Maximum Clique

Given G and integer k, in time $f(k)n^{O(1)}$ either

- find a $g(k)$-clique (for some unbounded nondecreasing function g) or
- correctly state that there is no k-clique.

Minimum Dominating Set

Given G and integer k, in time $f(k)n^{O(1)}$ either

- find a DS of size $g(k)$ or
- correctly state that there is no DS of size k.

Polynomial kernels

- Directed Feedback Vertex Set
- Multiway Cut (with arbitary number t of terminals)
- Planar Vertex Deletion

What about polynomial Turing kernels?

- k-Path

Directed Odd Cycle Transversal

Input: Directed graph G, integer k
Find: A set $X \subseteq U$ of at most k vertices such that $G - X$ has no directed cycle of odd length.

Generalizes

- Directed Feedback Vertex Set [Chen et al. 2008]
- Odd Cycle Transversal [Reed et al. 2004]
- Directed S-Cycle Transversal [Chitnis et al. 2012]

Square root phenomenon

Are there $2^{O(\sqrt{k}\cdot \text{polylog}(k))} n^{O(1)}$ time FPT algorithms for planar problems?

Some natural targets:

- Steiner Tree
- Directed Steiner Tree
- Directed Subset TSP

What about counting problems?

- k-path
- k-mathching
- k disjoint triangles
- k independent set

Disjoint paths/minor testing

- The best known parameter dependence for the k-disjoint paths problem and H-minor testing seems to be triple exponential. [Kawarabayashi and Wollan 2010] using [Chekuri and Chuzhoy 2014].
- For planar graphs, $2^{2^{\mathrm{poly}(k)}} n^{O(1)}$ algorithm. [Adler et al. 2011]
- Are there $2^{\mathrm{poly}(k)} n^{O(1)}$ time algorithms for planar or general graphs?