FPT suspects and tough customers: Open problems of Downey and Fellows

Fedor V. Fomin[1] and Dániel Marx

[1] Department of Informatics
University of Bergen, Norway
[2] Computer and Automation Research Institute
Hungarian Academy of Sciences (MTA SZTAKI)
Budapest, Hungary

Abstract. We give an update on the status of open problems from the book "Parameterized Complexity" by Downey and Fellows.

1 Introduction

Downey and Fellows' 1999 monograph [14] contains a list of open problems which strongly influenced the development of Parameterized Complexity in the following decade. Here we survey the current status of these problems.

Downey and Fellows partitioned their list of problems into two parts: "A Lineup of FPT Suspects" and "A Lineup of Tough Customers." While within the time some of the FPT suspects appeared to be tough customers and, vice versa, some of the tough customers turned to be not that tough, in our survey we decided to keep the original order and partition.

We do not provide definitions of classes FPT, XP, and W-hierarchy, referring to the book of Downey and Fellows [14], as well to more recent monographs of Flum and Grohe [18], and Niedermeier [37].

It is worthwhile to look back on this list now, more than 10 years later of its publication, and to try to see what we can learn from its history. An immediate and somewhat surprising observation is that with the exception of two problems, all the questions were resolved in the positive by fixed-parameter tractability results, even many of those which were classified as "tough customers" by Downey and Fellows. One can say that the algorithmic side of fixed-parameter tractability developed much more dramatically since 1999 than the complexity side. In the past 10 years, several fundamental and powerful techniques were introduced into the positive toolkit of fixed-parameter tractability (e.g., bidimensionality, iterative compression, algebraic techniques, inclusion-exclusion, various forms of randomization, etc.). On the other hand, while W[1]-hardness proofs got more streamlined over the years and we have now a better understanding of how to obtain hardness results for certain types of problems (e.g., for planar or bounded-treewidth problems), we do not have such a richness of standard techniques as in the case of algorithmic results. For most W[1]-hardness proofs, we still have to roll up our sleeves and reduce from Maximum Clique by constructing appropriate gadgets. If this trend continues, then we can expect to see further exciting developments in parameterized algorithmic techniques for several years. Apparently, the tools and theory of fixed-parameter tractability are even more deep and diverse than what Downey and Fellows expected in 1999 (and possibly what we see now). It is conceivable that it many cases, the main roadblock to understanding the complexity of a problem is not our limited ability to do W[1]-hardness proofs, but the fact that the right algorithmic technique for the problem is still waiting to be discovered.

2 A Lineup of FPT Suspects

Topological Containment FPT
Instance: An undirected graph G
Parameter: A graph H
Question: Is H topologically contained in G?

Graph H is topologically contained in G if a subdivision of H is a subgraph of G. The problem is in XP because one can guess all possible mapping of vertices of H into G and then for each guess apply the disjoint path algorithm of Robertson and Seymour [39]. The problem was shown to be in FPT by Grohe, Kawarabayashi, Marx, and Wollan in 2011 [23]. For every fixed undirected graph H, they gave an $O(|V(G)|^3)$ time algorithm for testing if a given graph G contains H topologically.

Immersion Order Test FPT
Instance: An undirected graph G
Parameter: A graph H
Question: Does H has an immersion in G?

An immersion of graph H into graph G is a mapping of vertices of H into vertices of G such that edges of H correspond to edge-disjoint paths of G. The problem is in FPT and solvable in $O(|V(G)|^3)$ time by reduction to Topological Containment [23].

Directed Feedback Vertex Set FPT
Instance: A directed graph G
Parameter: A positive integer k
Question: Is there a set S of k vertices such that each directed cycle of G contains a member of S?

The problem was shown to be in FPT by Chen, Liu, Lu, O'Sullivan, and Razgon in 2008 [6, 7]. The running time of the algorithm is $4^k k! n^{O(1)}$. It remains open if there exist a single exponential algorithm for Directed Feedback Vertex Set even on planar graphs. The existence of polynomial kernel is also open. The undirected variant of the problem, Feedback Vertex Set, received much more attention: the problem was proved to be in FPT by a simple combinatorial algorithm already in [13], it is known to be solvable in single exponential time [10, 25], and admits a polynomial kernel [43].

Planar Directed Disjoint Paths	Open
Instance: A directed planar graph G and k pairs $\langle r_1, s_1\rangle, \ldots, \langle r_k, s_k\rangle$ of vertices of G	
Parameter: k	
Question: Does G have k vertex-disjoint paths $P_1, \ldots, P_k$ with P_i running from r_i to s_i?	

The problem is open. Problem is in XP: Schrijver [41] showed that the problem is polynomial-time solvable for every fixed k. We remark that the paper of Schrijver is self-contained and in particular it does not use results from Graph Minors theory. The NP-hardness of the problem follows from the fact that even the undirected problem is NP-hard on planar graphs. For general graphs, the directed problem is NP-hard already for $k = 2$ [19].

Planar t-Normalized Weighted Satisfiability	FPT
Instance: A planar t-normalized formula X	
Parameter: A positive integer k	
Question: Does X have a satisfying assignment of weight k?	

A Boolean formula is t-normalized if it is of the form $\bigwedge\bigvee\bigwedge\ldots$ of literals with $t-1$ alternations of the $\bigwedge$ and $\bigvee$ quantifiers. For example, a 2-normalized formula is a CNF formula.

A CNF formula is planar if the bipartite graph of the formula (where one class is the set of clauses, the other class is the set of variables) is planar. However, it is not clear what the definition of a planar t-normalized formula should be and it is not defined in [14]. One obvious definition could be that the Boolean circuit describing the formula is planar. The problem with this definition is that "planar CNF formula" and "planar 2-normalized formula" are two different notions: the latter variant is more restrictive, as the Boolean circuit contains an output gate that is connected to all clauses. This suggests another, less restrictive, definition: a t-normalized formula is planar if the Boolean circuit describing the formula *with the output gate removed* is planar.

The problem is FPT even with the less restrictive definition of planarity. This follows from the fact that, for every fixed k and t, there is a first-order formula (over an appropriate planar structure) that expresses the existence of a weight-k satisfying assignment. Therefore, a powerful general result of Frick and Grohe [20] implies a linear-time algorithm for every fixed k and t. To construct this formula, one needs to express that there exists k variables such that the output gate (or more precisely, every input of the output $\bigwedge$ gate) is satisfied. As the formula is t-normalized, at most t quantifiers are needed to express that a gate is satisfied.

We sketch how a direct solution can be obtained by the standard layering and bounded-treewidth techniques on planar graphs ("Baker's shifting strategy"). The all-zero assignment determines a "standard" value v_g for every gate g. The key observation is that the only way g can have the opposite of v_g in some assignment if g is at distance at most t from a variable with value 1. This means that if we partition the graph into layers, then in every assignment of weight k, all but at most $(2t+1)k$ layers have the property that every gate has the standard value. By starting at some layer $i \leq (2t+1)k$ and forcing every $(2t+1)k+1$-st layer to take the standard value, the problem falls apart into independent subproblems, each having at most $(2t+1)k$ layers. Graphs with bounded number of layers are known to have bounded treewidth, hence the subproblems can be solved using standard techniques. Finally, our observation above implies that if there is a solution, then at least one choice of starting layer i is consistent with this solution, hence our algorithm finds a solution when considering this choice of i.

PLANAR MULTIWAY CUT	W[1]-hard

Instance: A weighted undirected planar graph G with terminals $\{x_1, \ldots, x_k\}$ and a positive integer M
Parameter: k
Question: Is there a set of edges of total weight $\leq M$ whose removal disconnects each terminal from all others?

The problem is known to be in XP: it can be solved in time $n^{O(k)}$ [27, 9] and more recently in time $2^{O(k)} \cdot n^{O(\sqrt{k})}$ [31]. The problem was shown to be W[1]-hard in 2011 [34]. Furthermore, assuming the Exponential Time Hypothesis [28], there is no $f(k)n^{o(\sqrt{k})}$ time algorithm for the problem.

For general graphs, the problem is NP-hard already for $k = 3$ [9]. When parameterized by the total weight of the solution, the problem is FPT on general graphs [35, 5, 24, 8] (the number of terminals can be arbitrary). The vertex-removal variant where the parameter is the total weight of the vertices to be deleted is also in FPT: the most recent algorithm of Cygan et al. [8] achieves the same running time for both versions.

3 A Lineup of Tough Customers

Fixed Alphabet Longest Common Subsequence (LCS)	W[1]-hard

Instance: k sequences X_i over an alphabet Σ of fixed size and a positive integer m
Parameter: k
Question: Is there a string $X \in \Sigma^*$ of length m that is a subsequence of each of the X_i?

Note that the characters in the subsequence X need not be consecutive in X_i. A simple $O(n^{k+1})$ time dynamic programming algorithm shows that the problem is in XP. When the size of the alphabet Σ is not bounded or when the parameter is $k + |\Sigma|$, the problem was known to be W[t]-hard for every $t \geq 1$ already in 1995 [3, 4]. Pietrzak in 2003 [38] showed that the problem is W[1]-hard parameterized by k, even if the alphabet is binary.

Bounded Hamming Weight Discrete Logarithm	Open

Instance: An n-bit prime p, a generator g of F_p^*, an element $a \in F_p^*$
Parameter: A positive integer k
Question: Is there a positive integer x whose binary representation has at most k 1's (that is, x has a Hamming weight of k) such that $a = g^x$?

Here F_p^* is the multiplicative group of non-zero integers modulo p. Element $g \in F_p^*$ is a generator of group F_p^* if for every element a, there exists an integer x with $a = g^x$. Note that there is a unique $1 \leq x \leq p - 1$ with $a = g^x$, but the problem definition does not insist that x should be less than p. To show that the problem is in XP, we need to argue that the representation of x is at most kn bits long (hence there are at most $(kn)^k$ different possibilities for x to try). See [17] for discussion and related problems.

The famous Discrete Logarithm problem is to find the unique $1 \leq x \leq p-1$ with $a = g^x$; the hardness of some cryptosystems are based on the assumed hardness of this problem. Because the problem definition does not require $x \leq p$, it is not completely obvious how the two problems relate to each other.

Crossing Number	FPT

Instance: An undirected graph G
Parameter: A positive integer k
Question: Is the crossing number of G is at most k?

The crossing number of a graph is the minimum number of edge crossings in a planar drawing of the graph (with the usual technical assumptions, such as no three edges cross at the same point). A graph is a planar graph if and only if its crossing number is 0. The problem asks if G can be drawn with at most k edge crossings. The problem was solved by Grohe in 2001 [22, 21], who showed that the problem is solvable in time $O(|V(G)|)^2$ for every fixed k. A linear-time algorithm is claimed in [30].

Downey and Fellows formulate the Crossing Number problem as “Can G be embedded in the plane with at most k edge*s* crossing?”, which can be interpreted as finding an embedding in which at most k edges participate in crossings. This is different from the classical definition of crossing number, but could be an interesting problem on its own right. A related problem is deciding if a graph is in the class “Planar+ke”, meaning that it can be made planar by removing at most k edges. A linear-time algorithm is claimed also for this problem by Kawarabayashi and Reed [30]. Note that having at most k edges participating in crossings and removing k edges to make the graph planar are two different problems: if a graph has an embedding where k edges participate in crossings, then the graph can be made planar by removing less than k edges (as there is no need to remove all the edges participating in crossings).

Minimum Degree Graph Partition — FPT
Instance: An undirected graph G
Parameter: Positive integers k and d
Question: Can $V(G)$ be partitioned into disjoint subsets $V_1, \ldots, V_m$ so that for $1 \leq i \leq m$, $|V_i| \leq k$ and at most d edges have exactly one endpoint in V_i?

Langston and Plaut [32] observed in 1998 that the graphs having such partitions for a fixed k and d are closed under taking immersions. Robertson and Seymour [40] proved that immersion is a well-quasi-ordering, which means that classes of graphs closed under immersion can be characterized by a finite number of forbidden immersed graphs. Together with the fact that the disjoint path algorithm of Robertson and Seymour [39] implies, for every fixed H, a polynomial-time algorithm for testing if H is immersed in G, it follows that the problem is in XP jointly parameterized by k and d. The result in 2011 that immersion testing is FPT [23] immediately implies that Minimum Degree Graph Partition is in (nonuniform) FPT.

Lokshtanov and Marx [33] showed in 2011 that the problem is in FPT parameterized by k or by d by establishing a more general result. In the (μ, p, q)-Partition problem, the task is to find a partition of the vertices where each cluster C satisfies the requirements that at most q edges leave C and $\mu(C) \leq p$. It was shown in [33] that when μ is one of the following functions—number of nonedges in the cluster, maximum degree of nonedges in the cluster, number of vertices in the cluster—(μ, p, q)-Partition can be solved in time $2^{O(p)}n^{O(1)}$ and in time $2^{O(q)}n^{O(1)}$, i.e., the problem is fixed-parameter tractable parameterized by p or by q.

Short Cheap Tour FPT
Instance: A graph G, integer S, and edge weighting $w\colon E(G) \to \mathbb{Z}$
Parameter: A positive integer k
Question: Is there a tour through at least k nodes of G of cost at most S?

As observed by Fellows [16] in 2001, the problem is FPT by a simple reduction to finding a minimum weight cycle of length exactly k, which can be solved by color coding [2]. We sketch the reduction. Let G' be a complete graph on the same set of vertices as G, and let the weight of edge uv be the length of the shortest path between u and v in G. It is easy to see that G has a tour visiting at least k nodes of cost at most S if and only if G' has a cycle of length *exactly* k of cost at most S.

The variant of the problem where we ask that the cost of the tour is exactly S is W[1]-hard [12].

Polymatroid Recognition Open
Instance: A k-polymatroid M
Parameter: A positive integer k
Question: Is M hypergraphic?

Let E be a finite set. A polymatroid is a function $\rho : 2^E \to \mathbb{Z}$ with the following properties:

1. $\rho(\emptyset) = 0$,
2. $\rho(A) \leq \rho(B)$ for every $A \subseteq B \subseteq E$, and
3. $\rho(A) + \rho(B) \geq \rho(A \cap B) + \rho(A \cup B)$.

A k-polymatroid is a polymatroid with $\rho(e) \leq k$ for every $e \in E$. Given a hypergraph H with vertex set V and edge set E, the hypergraphic polymatroid of H is a function $\chi_H : 2^E \to \mathbb{Z}$ defined by

$$\chi_H(A) = |\overline{A}| - \kappa(H|A),$$

where $\overline{A}$ is the set of vertices contained in the edge set A, and $\kappa(H|A)$ is the number of components of the hypergraph H restricted to A (see [45] for more details). A polymatroid is hypergraphic, if it is the hypergraphic polymatroid of a hypergraph.

A word of caution should be said on how the polymatroid is given in the input. One possibility is that it is given by an oracle, but then the problem does not fit the framework of complexity theory defined by problems as languages (but it is still an interesting question if $f(k) \cdot n^{O(1)}$ oracle calls are sufficient for the problem).

Chain Minor Ordering	Open
Instance: A finite poset Q	
Parameter: A finite poset P	
Question: Is P a chain minor of Q?	

Let $P = (V, <)$ be a poset. A chain is a sequence of elements $x_1 < x_2 < \ldots < x_n$. We say that $P = (V, <)$ is a chain minor $P' = (V', <)$ if there is a partial mapping $\rho : V' \to V$ with the following property: for every chain C of P, there is a chain C' of P' such that ρ restricted to C' is an isomorphism of chains from C' to C. Gustedt [26] showed that the problem is in XP and that the chain minor relation is a well-quasi-ordering. The problem remains open.

Short Generalized Hex	Open
Instance: An undirected graph G with two distinguished vertices v_1 and v_2	
Parameter: A positive integer k	
Question: Does player one have a winning strategy of at most k moves in Generalized Hex?	

In Generalized Hex two players play on a graph with white and black pebbles. Player one plays with white and player two with black pebbles. Player one starts by placing a white pebble on a vertex of G. Then alternately players make moves, at each move a pebble is placed on an occupied vertex. Player one wins if he can construct a path of white vertices from v_1 to v_2.

To the best of our knowledge, the problem remains open. Downey and Fellows [14] proposed that the problem is a good candidate for AW[*]-completeness. Towards this goal, Allan [42] showed that the problem is in AW[*].

Jump Number	FPT
Instance: A poset P	
Parameter: A positive integer k	
Question: Is the jump number of P at most k?	

Given a finite partially ordered set (or poset) $P = (V, <_P)$, let $L = (V, <_L)$ be a linear extension of P, that is a total order on the same ground set V of P, such that each couple of elements $u, v \in V$ for which $u <_P v$ implies $u <_L v$. A consecutive pair (v_i, v_{i+1}) of elements in L is a jump or setup of L if $v_i \not<_P v_{i+1}$. The jump number of P is the minimum number of jumps in L, where minimum is taken over all the linear extensions L of P.

The problem was shown to be in XP by El-Zahar and Schmerl [15] in 1984. McCartin showed in 2001 [36] that the problem is in FPT.

Polynomial Product Identity	Open

Instance: Two sets of k multivariate polynomials p_i and q_i for $i = 1, \dots, k$
Parameter: k
Question: Does the following identity hold?

$$\prod_{i=1}^{k} p_i = \prod_{i=1}^{k} q_i?$$

The polynomials in the input are given by listing the monomials with nonzero coefficients. Note that there is no bound on the number of variables or on the degree of the polynomials. By multiplying out each product, we get at most n^k monomials and we can compare the two sides to test for equality. Therefore the problem is in XP.

As discussed in [29, Section 4.3], the Schwartz-Zippel Lemma provides a way of solving the problem in randomized polynomial time and therefore it is in randomized FPT. Thus the problem is unlikely to be W[1]-hard. It could still be a nontrivial question if the problem is in deterministic FPT. Answering this question may tell us something interesting about the tradeoff between randomness and running time.

Shortest Vector	Open

Instance: A basis $X = \{x_1, x_2, \dots, x_n\} \subset \mathbb{Z}^n$ for a lattice $\mathcal{L}$
Parameter: A positive integer k
Question: Is there a non-zero vector $x \in \mathcal{L}$, such that $\|x\|^2 \le k$?

Here $\|x\|$ denotes the Euclidean (ℓ_2) norm of $x = (a_1, \dots, a_b)$, defined as $\sqrt{\sum_{i=1}^{n} a_i^2}$. The problem was shown to be NP-hard hard under randomized reduction by Ajtai in 1998 [1], settling a longstanding open problem. The problem is in XP: every vector x with $\|x\|^2 \le k$ contains at most k nonzero coordinates. It could be interesting to investigate the problem for other ℓ_p norms as well.

Even Set	Open

Instance: An undirected red/blue bipartite graph $G = (\mathcal{R}, \mathcal{B}, E)$
Parameter: A positive integer k
Question: Is there a non-empty set of at most k vertices $R \subseteq \mathcal{R}$, such that each member of $\mathcal{B}$ has an even number of neighbors in R?

Open. The exact version of the problem, where $|R| = k$, is W[1]-hard [11]. Vardy [44] proved the NP-completeness of the problem in 1997, settling a longstanding open problem. There are other equivalent ways of stating the problem, showing that this problem appears naturally in many contexts:

- Given a hypergraph H, is there a nonempty set S of at most k vertices, such that $|e \cap S|$ is even for every hyperedge e?
- Given a matrix A over the two-element field $GF[2]$, is there a nonzero vector $\mathbf{x}$ having at most k nonzero coordinates and satisfying $A\mathbf{x} = 0$?
- Given a binary linear code defined by a matrix A over $GF[2]$, are there two codewords with Hamming-distance at most k?
- Given a binary matroid represented by a matrix A over $GF[2]$, does it have a cycle of length at most k?

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