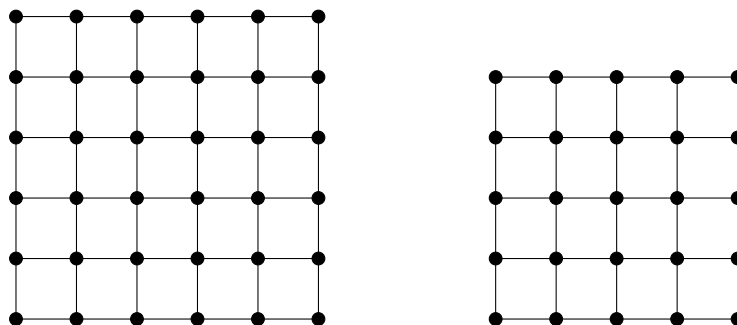
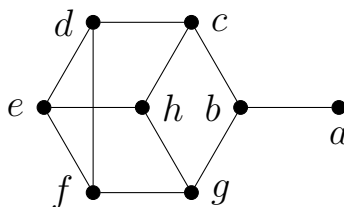


- For which of the values  $r = 1, 2, \dots, 9$  does it hold that every simple  $r$ -regular graph on 10 vertices contains an Euler circuit? (A graph is  $r$ -regular if the degree of each of its vertices is  $r$ .)
- Let the vertex set of the complete graph  $K_{10}$  be  $V(K_{10}) = \{1, 2, \dots, 10\}$ . We obtain the graph  $G$  by deleting the edges  $\{1, 2\}, \{1, 3\}, \{2, 3\}$  and  $\{4, 5\}, \{4, 6\}, \{5, 6\}$  from  $K_{10}$ . Determine  $\chi(G)$ , the chromatic number of  $G$ .
- Decide whether the graphs below contain a Hamilton cycle or not.



- A graph on 20 vertices has 18 edges and two components. Show that the graph is planar.
- Determine the edge-chromatic number of the graph below.



- (\*) In a simple bipartite graph on 20 vertices the degree of each vertex is either 5 or 6. Show that the graph contains a perfect matching.

Total work time: 90 min.

The full solution of each problem (including explanations) is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Notes and calculators (and similar devices) cannot be used.