1. (MT+’11) For which values of $k$ is it true that the following graph is
   a) $k$-edge-connected;
   b) $k$-vertex-connected?

2. Determine the vertex- and edge connectivity numbers ($\kappa(G)$ and $\lambda(G)$) of the following graphs:
   a) the graph consisting of the vertices and edges of a cube,
   b) the complete bipartite graph $K_{m,n}$, where $m \geq n$.

3. The vertices of an 18-vertex graph $G$ can be divided into 3 classes of six vertices each, in such a way
   that 2 vertices are adjacent if and only if they are in different classes. Determine the largest integer
   $k$ for which $G$ is $k$-vertex-connected ($\kappa(G)$), and the largest integer $l$ for which $G$ is $l$-edge-connected
   ($\lambda(G)$).

4. Show that a $k$-(vertex-)connected graph $G$ on $n$ vertices has at least $kn/2$ edges.

5. Prove that an $n/2$-(vertex-)connected graph on $n$ vertices contains a Hamilton cycle.

6. Construct a simple graph which is 2-vertex-connected, 3-edge-connected and has minimum degree
   4.

7. (MT’14) We connect two disjoint complete graphs on 5 vertices with 3 edges, in such a way that
   the resulting graph $G$ is simple. Is it true in all cases that $G$ is
   a) 3-(vertex)-connected;
   b) 3-edge-connected?

8. (MT’17) A simple graph on 10 vertices has 40 edges. Determine the largest integer $k$ for which $G$
   is surely $k$-vertex-connected.

9. Show that if a graph is 3-(vertex-)connected, then it contains a cycle of even length.

10. (MT’07) Let $G$ be a 3-(vertex-)connected graph with 100 vertices and let $x, y \in V(G)$ be two
    different vertices. Show that there is a path from $x$ to $y$ whose length (i.e. the number of edges in
    it) is not greater than 33.

11. a) Let $G$ be a $k$-connected graph, and $G'$ be a graph obtained by adding a new vertex of degree at
    least $k$ to $G$. Show that if $G'$ is a simple graph, then it is $k$-(vertex-)connected as well.
    b) Let $G$ be a $k$-connected graph, and $A = \{a_1, \ldots, a_k\}$ and $B = \{b_1, \ldots, b_k\}$ be two disjoint point
    sets in it. Prove that there are $k$ (completely) vertex-disjoint paths in $G$ connecting $A$ and $B$. 