Introduction to the Theory of Computing 1.
Re-retake of the Second Midterm Test

2022. December 19.

1. We call a vector $\underline{v}$ in $\mathbf{R}^5$ *semisymmetric* if both the sum of its first and last coordinates and the sum of its second and fourth coordinates are equal to the third (middle) coordinate. (E.g. the vector $(1,3,8,5,7)^T$ is a semisymmetric vector.) Determine the dimension of the subspace V of $\mathbf{R}^5$ consisting of semisymmetric vectors. (For the solution you don't need to show that V is in fact a subspace.)

2. Solve the following system of linear equations for each value of the parameter p.
$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 6 \\ 2x_1 + 6x_2 + p \cdot x_3 &= 12 \\ x_1 + 8x_2 + 6x_3 &= 6 \end{aligned}$$

3. Let A be the matrix below. Evaluate the determinant of $A \cdot A^T$
$$A = \begin{pmatrix} 2 & 4 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 3 \end{pmatrix}$$

4. The matrix below is the inverse of the matrix B. Decide whether the matrix B^2 has an inverse, and if yes then determine it (where B^2 denotes the matrix $B \cdot B$).
$$B^{-1} = \begin{pmatrix} 3 & 5 \\ 2 & 3 \end{pmatrix}$$

5. All we know of the 5×5 matrix C is that it has exactly 3 non-zero entries. Determine all possible values of the rank of C.

6. * The determinant of the 10×10 matrix D is 0. Moreover, no matter how we change one entry of D, the determinant of the matrix obtained is always 0 as well. Show that the rank of D is at most 8.

Please work on stapled sheets only, and submit all of them at the end of the midterm, including drafts. Write your name on every sheet you work on, and write your Neptun code on the first page.

You have 90 minutes to work on the problems. Each of them is worth 10 points. The problem marked with an * is supposed to be more difficult.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators (or similar devices) cannot be used.

Please turn over for the first midterm.