

Introduction to the Theory of Computing I.

Second Midterm Test

2021. December 2.

1. Evaluate the first determinant below *using the original definition*. (So don't use any properties of the determinant, or theorems about it during the solution, but determine the value using the definition only.)

$$\begin{vmatrix} 7 & 2 & 9 & 6 & 4 \\ 3 & 0 & 0 & 0 & 5 \\ 6 & 0 & 8 & 1 & 7 \\ 5 & 0 & 0 & 0 & 8 \\ 9 & 0 & 2 & 0 & 4 \end{vmatrix} \qquad \begin{vmatrix} 0 & -1 & -2 & -3 & -4 \\ 1 & 0 & 5 & 6 & 7 \\ 2 & -5 & 0 & -8 & -9 \\ 3 & -6 & 8 & 0 & 10 \\ 4 & -7 & 9 & -10 & 0 \end{vmatrix}$$

2. Evaluate the second determinant above (by any method).
3. Let A and B be the matrices below and let $C = B^{-1} \cdot A \cdot B$. Determine the matrices A^{100} and C^{100} .

$$A = \begin{pmatrix} 1 & -4 & -2 \\ 0 & -3 & -2 \\ 0 & 4 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(For the full solution you don't have to show that B^{-1} exists. For a matrix M , M^{100} is the product with 100 factors, each of whose is M .)

4. The upper left entry of the 10×10 matrix A is 2, all the other entries on its main diagonal are 1, and the rest of its entries (i.e. off the main diagonal) are 2. (So 9 of the entries of A are 1, and 91 entries of it are 2.) Decide whether A is invertible or not, and if yes, determine its inverse.
5. Determine the rank of the matrix below for all the values of the parameter p .

$$A = \begin{pmatrix} 5 & 15 & -30 & 20 \\ 1 & 0 & -21 & 10 \\ -2 & -8 & p & 2p - 8 \\ 2 & 9 & 3 & p \end{pmatrix}$$

6. * We know of a system of linear equations that is has a solution for which the sum of the variables is 2020, but it has no such solution for which the sum of the variables is 2021. Can we decide based on this information whether the same system of linear equations has a solution for which the sum of the variables is 2022?

Please work on stapled sheets only, and submit all of them at the end of the midterm. Write your name on every sheet you work on, and write your Neptun code and the number of the group you are registered to in Neptun (A1 or A2) on the first page.

You have 90 minutes to work on the problems, each of them is worth 10 points. To obtain a signature you have to achieve at least 18 points on each of the two midterm tests, and the total points from the two midterms should be at least 48.

The details of the solutions must be explained, giving the result only is not worth any points. Notes, calculators or any additional tools cannot be used. The problem marked with an * is supposed to be more difficult.