Introduction to the Theory of Computing I.
Second repeat of the midterm
21 December, 2020

1. Before the holidays a candy shop decides to sell the leftover candies for a lower price in pre-boxed holiday packages. The estimated number of the remaining candies is between 400 and 600. If 12 candies are packed in every box, then there are 7 candies left, while if they try to create giant packages with 50 candies in each of them, then in the last box one candy is missing. How many candies are left when 16 candies are packed in every box?

2. Assume that $n = 987654321$. Apply the appropriate algorithm that was told in the lecture to determine the greatest common divisor of $98n + 27$ and $76n + 21$. (Document every step of the algorithm.)

3. A ray of light coming from the source $P(5;\, -6;\, 28)$ goes through the point $Q(14;\, 15;\, 16)$ and also passes through the plate glass with the equation $x + 3y - z = 1$ (not necessarily in this order).

a) Determine the intersection of the ray and the glass.

b) Decide if the points P and Q are on the same side of the glass or not.

(We can assume that the light travels in a straight line and it passes through the glass without changing its direction.)

4. Let V be the set of those vectors in $\mathbb{R}^4$ whose coordinates (ordered by position) form a geometric progression, i.e. the quotient of the consecutive coordinates is the same (if we divide the coordinate with the greater index by the coordinate with the smaller index). Decide if V is a subspace of $\mathbb{R}^4$ or not.

5. Let us consider the following vectors in $\mathbb{R}^4$:

$$\underline{u} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \ \underline{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \ \underline{w} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \underline{a} = \begin{pmatrix} p \\ p+1 \\ p+2 \\ 2p \end{pmatrix}.$$

Determine the possible values of p for which the statement $\underline{a} \in \operatorname{span}\{\underline{u}, \underline{v}, \underline{w}\}$ holds.

6*. Let $n = 898989\ldots89$ be the positive integer with 134 digits whose $(2k-1)$th digits are 8 and whose $(2k)$th digits are 9 (counted from the left, where $k = 1, \ldots, 67$). What is the remainder of n when divided by 67.

During the midterm any written lecture notes or books can be used, but cooperation with another person is forbidden. Solutions built upon the usage of a calculator or a computer program are not accepted. The detailed computations that lead to the conclusions form important parts of the solutions, hence they have to be submitted as well.

You have 90 minutes to work on the problems and extra 25 minutes to scan and upload your solutions. Each problem is worth 10 points. To obtain a signature you have to achieve at least 24 points. The problem marked with an * is supposed to be more difficult.