

Introduction to the Theory of Computing 1.

Repeated First Midterm Test

2019. December 16.

1. How many positive integers are there which are not greater than 504 and have a multiple which gives 1 as a remainder when divided by 504?
2. Determine all the integers between 1 and 1000 for which $n+10$ divided by 36, and $n - 10$ divided by 38 both give a remainder of 1.
3. Does the plane of equation $5x - 3y + 2z = 1$ have a point P for which the points P , $Q(5, 9, 11)$ and $R(13, 7, 7)$ are on one line? If yes, then determine all such points P .
4. Let $\underline{u} = (1, -2, 0, 0)^T$, $\underline{v} = (0, 1, -4, 0)^T$ and $\underline{w} = (0, 0, 5, -5)^T$ be vectors in \mathbf{R}^4 . The first two coordinates of the vector \underline{a} in the generated subspace $\langle \underline{u}, \underline{v}, \underline{w} \rangle$ are both 1. Determine the sum of the coordinates of \underline{a} .
5. Determine those values of the parameter p for which the vectors $(1, -1, 1, 0)^T$, $(0, 1, -2, 1)^T$ and $(-1, 0, 1, p)^T$ are linearly independent.
6. * Is there an integer n for which $n^4 + 1$ is divisible by 101?

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.

Notes and calculators (and similar devices) cannot be used. The question denoted by an * is supposed to be more difficult.