Introduction to the Theory of Computing 1.
Second Repeat of the First Midterm Test
2018. December 17.

1. Determine all the three-digit integers whose last two digits in both of the numerical system of base 4 and in the numerical system of base 5 are 11 as well.

2. * Determine the sum of those positive numbers which are less than 630 and are relative prime to 630.

3. The code written in C below calculates the sum of the digits of the positive integer n (written in the decimal system). Suppose that the computer uses the "normal" basic operations (addition, subtraction, multiplication, division,...). Determine whether the algorithm is polynomial or not. (`floor(n/10.0)` gives the lower integer part of $\frac{n}{10}$ back.)

```
x = 0; y = 0;
while (n > 0) {
   x = floor(n/10.0);
   y =y+n-10*x;
   n = x;
}
printf(``Result: %d'', y);
```

4. The ray of light starting from the light source at the point $P(3, 17, 27)$ falls perpendicularly on the plane mirror of equation $3x - y - 2z = 8$. Determine the point of incidence. (We can suppose that the light is spread in a straight line.)

5. Let $\underline{a} = (1, 4, 1)^T$, $\underline{b} = (2, 1, -1)^T$, $\underline{c} = (12, 13, -3)^T$ and $\underline{d} = (7, 7, -2)^T$ be vectors in $\mathbf{R}^3$. Do they form a generating system in $\mathbf{R}^3$?

6. Let the vectors $\underline{a}, \underline{b}, \underline{c} \in \mathbf{R}^n$ be linearly independent. Does it follow that the vectors $3\underline{a} - \underline{b}$, $2\underline{a} + \underline{c}$ and $4\underline{b} - 3\underline{c}$ are always linearly independent?

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit.
Notes and calculators (and similar devices) cannot be used. The question denoted by an * is supposed to be more difficult.