Introduction to Computer Science I.
Second Repeat of the Second Midterm Test

2017. December 18.

1. All that we know of the subspace V of $\mathbf{R}^4$ is that it contains each of the vectors $(1,0,1,0)^T$, $(1,0,0,1)^T$, $(0,1,0,1)^T$ and $(0,1,1,0)^T$. Can we determine the dimension of V from this?

2. Evaluate the determinant below for all real values p.

$$\begin{vmatrix} p & 2p & p & 3p \\ 3 & 9 & 3 & 6 \\ 1 & 2 & 7 & 1 \\ 3 & 7 & 8 & 4 \end{vmatrix}$$

3. Determine for which values of the parameter p the system of equations below is consistent. If it has solutions, then determine all of them.

$$\begin{array}{rcl} px_1 + 2px_2 + px_3 & = & 3p \\ 3x_1 + 9x_2 + 3x_3 & = & 6 \\ x_1 + 2x_2 + 7x_3 & = & 1 \\ 3x_1 + 7x_2 + (p+15) \cdot x_3 & = & 4 \end{array}$$

4. Let A be the matrix below. Is there such a matrix B for which BA is the 3×3 identity matrix?

$$A = \begin{pmatrix} 2 & 4 & 6 \\ 3 & 7 & 10 \end{pmatrix}$$

5. Determine the rank of the matrix below depending on the parameter x.

$$\begin{pmatrix} 3 & 6 & 9 \\ 2 & 4 & x \\ 2 & 5 & x \\ 2 & 6 & x \end{pmatrix}$$

6. We call a 4×4 matrix *nicely extendable*, if we can add a new column and row to it to get a 5×5 invertible matrix. Determine whether the following statements are true or not:
   a) If a 4×4 matrix is nicely extendable then its rank is at least 3.
   a) If the rank of a 4×4 matrix is at least 3 then it is nicely extendable.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.