Introduction to Computer Science I.
Repeated Second Midterm Test

2017. December 11.

1. Let V be the subspace of $\mathbf{R}^4$ consisting of the vectors $\underline{x} = (x_1, x_2, x_3, x_4)^T$, whose coordinates satisfy the equation $x_4 = x_1 + 2x_2 - 3x_3$. Determine a basis in V containing the vector $\underline{v} = (1, 1, 1, 0)^T$.

2. Evaluate the determinant below for all real values p.

$$\begin{vmatrix} p & 1 & 3 & 7 \\ 1 & p & 8 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & p & p \end{vmatrix}$$

3. Determine for which values of the parameter p the system of equations below is consistent. If it has solutions, then determine all of them.

$$\begin{aligned} x_1 + 3x_2 + 6x_3 + 2x_4 &= 5 \\ x_1 + 5x_2 + 8x_3 + 4x_4 &= 7 \\ 2x_1 + 6x_2 + (p+12) \cdot x_3 + (p+5) \cdot x_4 &= 10 \end{aligned}$$

4. Let A be the matrix below. Determine a matrix B for which AB is the 2×2 identity matrix.

$$A = \begin{pmatrix} 2 & 4 & 7 \\ 3 & 7 & 10 \end{pmatrix}$$

5. Determine the rank of the matrix below depending on the parameter p.

$$\begin{pmatrix} 3 & 6 & 12 & 9 \\ 2 & 4 & x & 3x \\ 1 & 5 & 6 & 9 \end{pmatrix}$$

6. A system of 10 linear equations with 10 unknowns is given by its augmented matrix. We know that the system has infinitely many solutions.
a) Is it true that we can always get a system of linear equations with a unique solution by changing one entry of the augmented matrix?
b) Is it true that we can always get a system of linear equations with no solutions by changing one entry of the augmented matrix?

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.