Introduction to Computer Science I.
Second Midterm Test
2017. November 30.

1. The vectors $\underline{a}, \underline{b}, \underline{c}, \underline{d}$ form a basis of $\mathbf{R}^4$. Determine the dimension of the subspace generated by the vectors $\underline{a}+\underline{b},\ \underline{c}+\underline{d},\ \underline{a}+\underline{c},\ \underline{b}+\underline{d}$.

2. Determine for which values of the parameter p the system of equations below is consistent. If it has solutions, then determine all of them.

$$\begin{aligned} x_1 + 3x_2 + 4x_3 &= 5 \\ 2x_1 + 9x_2 + 14x_3 &= 13 \\ x_1 + p \cdot x_2 + p \cdot x_3 &= 4 \end{aligned}$$

3. Evaluate the determinant below *using the original definition.* (So don't use any properties of the determinant, or theorems about it during the solution, but determine the value using the definition only.)

$$\begin{vmatrix} 0 & 0 & 1 & 2 & 5 \\ 3 & 0 & 6 & 8 & 9 \\ 0 & 0 & 5 & 0 & 0 \\ 5 & 4 & 7 & 3 & 2 \\ 0 & 0 & 2 & 0 & 1 \end{vmatrix}$$

4. a) Determine all the values p for which the matrix below has an inverse.
b) Determine the upper left entry of the inverse matrix if $p = 5$.

$$\begin{pmatrix} 2 & 4 & 0 \\ 5 & 10 & p \\ 4 & 9 & 1 \end{pmatrix}$$

5. Determine the rank of the following matrix depending on the parameters p and q.

$$\begin{pmatrix} p & 1 & p & 1 \\ 0 & 0 & 1 & 1 \\ 1 & q & 1 & q \end{pmatrix}$$

6. The rank of the 4×5 matrix A is 4. Show that A has at least six invertible 2×2 submatrices.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.