Introduction to Computer Science I.
Repeated First Midterm Test

2017. December 11.

1. Determine all the four-digit integers which give a remainder of 3 when divided by 51, furthermore if we multiply them by 17, then the last two digits of the product are 15.

2. Determine the remainder we get if we divide $73^{37} + 37^{73}$ by 108.

3. Use the algorithm we learnt to determine the remainder we get if we divide 5^{85} by 155.

4. Does the plane through the points $A(-1,-2,1)$, $B(3,1,3)$ and $C(7,6,3)$ contain a point which is on the y axis? If yes, then which is it?

5. Let $\underline{u} = (0,0,1,2)^T$, $\underline{v} = (0,1,2,5)^T$ and $\underline{w} = (1,2,4,11)^T$ be vectors in $\mathbf{R}^4$. Determine $\langle \underline{u}, \underline{v}, \underline{w} \rangle$, the subspace generated by them. (That is, give a (system of) equation(s), satisfied by the vectors in $\langle \underline{u}, \underline{v}, \underline{w} \rangle$.)

6. Suppose that for the vectors $\underline{v}_1, \underline{v}_2, ..., \underline{v}_{10}, \underline{w}$ in $\mathbf{R}^n$ it holds that $\underline{v}_1, \underline{v}_2, ..., \underline{v}_{10}$ are linearly independent, but $\underline{v}_1, \underline{v}_2, ..., \underline{v}_{10}, \underline{w}$ are linearly dependent, and $\underline{w} \neq \underline{0}$. Show that there is an index $1 \leq i \leq 10$ and a scalar $\alpha \neq 0$, such that the vectors $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_{i-1}, \underline{v}_i + \alpha \cdot \underline{w}, \underline{v}_{i+1}, \ldots, \underline{v}_{10}$ are linearly dependent.

The full solution of each problem is worth 10 points. Show all your work! Results without proper justification or work shown deserve no credit. Calculators (or other devices) are not allowed to use.